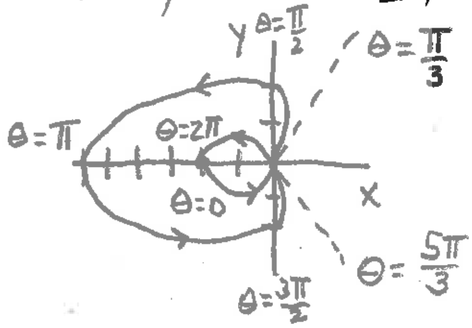
MATH 237 Sochacki Home Work 1 Name KEY WEST
Sept. 20, 2016 Take Home 50 Points (Print) (1 point)

All necessary work must be shown for credit. Your work must represent the question asked. You may NOT use computers. You may use your notes or text. Your work must be neat or I will not grade your work. You may discuss this assignment with others, but all work turned in must be your own work.

I have neither received nor given help on this exam. Don Key
(Signature) (1 point)

1. Consider the polar curve $r = 2 - 4\cos\theta$. (a) Give the integral in simplified form that gives the arc length of the smallest loop of the curve (you do not have to integrate). (b) Give the area of the region inside the curve.
(c) Give the equation of the tangent line at $\theta = \frac{\pi}{4}$. (6 points)

$r = 0,\ \cos\theta = \frac{1}{2},\ \theta = \frac{\pi}{3}, \frac{5\pi}{3}$



(a) $S = 2\int_0^{\frac{\pi}{3}} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta = 2\int_0^{\pi/3} \sqrt{(2-4\cos\theta)^2 + (4\sin\theta)^2}$

$= 2\int_0^{\frac{\pi}{3}} \sqrt{20 - 16\cos\theta}\, d\theta = 4\int_0^{\frac{\pi}{3}} \sqrt{5 - 4\cos\theta}\, d\theta$

(b) $A = 2\left[\frac{1}{2}\int_{\frac{\pi}{3}}^{\pi} (2-4\cos\theta)^2\, d\theta\right]$

$= \int_{\frac{\pi}{3}}^{\pi} (4 - 16\cos\theta + 16\cos^2\theta)\, d\theta$

$= (4\theta - 16\sin\theta)\Big|_{\frac{\pi}{3}}^{\pi} + 8\int_{\frac{\pi}{3}}^{\pi} (1 + \cos 2\theta)\, d\theta$

$= 4\pi - \left(\frac{4\pi}{3} - 8\sqrt{3}\right) + 8\left(\theta + \frac{1}{2}\sin 2\theta\right)\Big|_{\frac{\pi}{3}}^{\pi}$

$= \frac{8\pi}{3} + 8\sqrt{3} + 8\left(\pi - \left(\frac{\pi}{3} - \frac{1}{2}\frac{\sqrt{3}}{2}\right)\right)$

$= 8\pi + 6\sqrt{3}$

$X = r\cos\theta = (2-4\cos\theta)\cos\theta = 2\cos\theta - 4\cos^2\theta$

$Y = r\sin\theta = (2-4\cos\theta)\sin\theta = 2\sin\theta - 4\cos\theta\sin\theta$

$\frac{dx}{d\theta} = -2\sin\theta + 8\cos\theta\sin\theta \qquad \frac{dx}{d\theta}\Big|_{\theta=\frac{\pi}{4}} = 4 - \sqrt{2}$

$\frac{dy}{d\theta} = 2\cos\theta + 4\cos^2\theta - 4\sin^2\theta \qquad \frac{dy}{d\theta}\Big|_{\theta=\frac{\pi}{4}} = \sqrt{2}$

c) $x\left(\frac{\pi}{4}\right) = \sqrt{2} - 2 \quad y\left(\frac{\pi}{4}\right) = \sqrt{2} - 2 \quad y - (\sqrt{2}-2) = \frac{\sqrt{2}}{4-\sqrt{2}}\left(x - (\sqrt{2}-2)\right)$

2. Give the equation of the line whose direction is given by the line $\frac{x+1}{3} = \frac{1-y}{4} = 2z$ and who passes through the point (1,-1,2). (6 points)

$\frac{x+1}{3} = \frac{1-y}{4} = 2z = t$

$x = 3t + 1,\ -y = 4t - 1,\ z = \frac{1}{2}t$

$y = -4t + 1$

$\Delta x = 3,\ \Delta y = -4,\ \Delta z = \frac{1}{2}$

$\ell(t) = (1,-1,2) + t\langle 3, -4, \frac{1}{2}\rangle$

$= \langle 1 + 3t, -1 - 4t, 2 + \frac{1}{2}t\rangle$

$x = 1 + 3t$

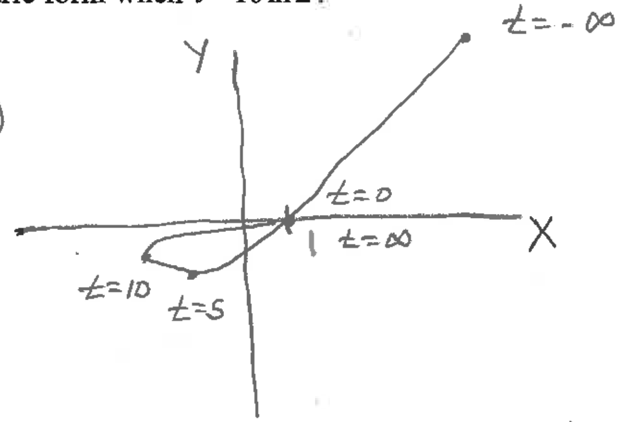
$y = -1 - 4t$

$z = 2 + \frac{1}{2}t$

3. Consider the parametric curve $\left(1-te^{-\frac{t}{10}},-te^{-\frac{t}{5}}\right)$. (a) Sketch the graph of this curve for $-\infty<t<\infty$ using limits and derivatives. (b) Give the equation of the tangent line in parametric form when $t=10\ln 2$.

(c) Give $\frac{d^2y}{dx^2}$ when t = 1. (6 points)

(a)



$X = 1-te^{-\frac{t}{10}} \quad Y = -te^{-\frac{t}{5}}$

$X' = -e^{-t/10} + \frac{t}{10}e^{-\frac{t}{10}}, \quad Y' = -e^{-\frac{t}{5}} + \frac{1}{5}te^{-\frac{t}{5}}$

$X' = \frac{-1}{10}e^{-t/10}(10-t), \quad Y' = \frac{-1}{5}e^{-t/5}(5-t)$

$X'' = \frac{1}{100}e^{-t/10}(10-t) + \frac{1}{10}e^{-t/10}$

$Y'' = \frac{1}{25}e^{-t/5}(5-t) + \frac{1}{5}e^{-t/5}$

$X'(10\ln 2) = \frac{1}{2}(\ln 2-1) \quad Y'(10\ln 2) = \frac{2\ln 2-1}{4}$

t	X	Y
$-\infty$	∞	∞
0	1	0
5	$1-5e^{-\frac{1}{2}}$	$-5e^{-1}$
10	$1-10e^{-1}$	$-10e^{-2}$

t	X	Y
∞	1	0
$10\ln 2$	$1-5\ln 2$	$\frac{-5}{2}\ln 2$

X' − | X' + (t, 10)

Y' − | Y' + (t, 5)

(b) $T(t) = \left(1-5\ln 2, \frac{-5}{2}\ln 2\right) + t\left(\frac{\ln 2-1}{2}, \frac{2\ln 2-1}{4}\right)$

(c) $$\frac{d^2y}{dx^2} = \frac{X'(1)Y''(1)-Y'(1)X''(1)}{X'(1)^3}$$

$$= \frac{\left(\frac{-9}{10}e^{-\frac{1}{10}}\right)\left(\frac{9}{25}e^{-1/5}\right)-\left(\frac{-4}{5}e^{-\frac{1}{5}}\right)\left(\frac{19}{100}e^{-\frac{1}{10}}\right)}{\left(\frac{-9}{10}e^{-1/10}\right)^3}$$

4. Let $\vec{u}=\langle 2,-3,2\rangle, \vec{v}=3\vec{i}-4\vec{k}$ give the answer to the following problems. (6 points)

(a) A unit vector in the $\vec{u}$ direction is

$\|\bar{u}\| = \sqrt{2^2+(-3)^2+2^2} = \sqrt{17}$

$\bar{w} = \frac{1}{\sqrt{17}}\bar{u} = \frac{1}{\sqrt{17}}\langle 2,-3,2\rangle = \frac{2}{\sqrt{17}}\bar{i} - \frac{3}{\sqrt{17}}\bar{j} + \frac{2}{\sqrt{17}}\bar{k}$

is a unit vector.

(b) The angle between $\vec{u}$ and $\vec{v}$ is

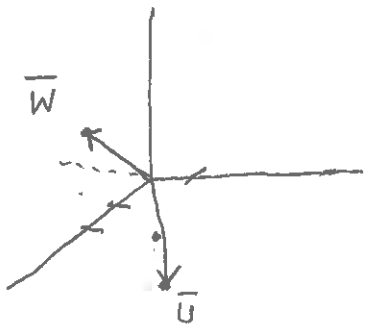
$\|\bar{v}\| = \sqrt{3^2+4^2} = 5$

$\cos\theta = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\cdot\|\bar{v}\|} = \frac{6-8}{\sqrt{17}\cdot 5} = \frac{-2}{5\sqrt{17}}$

$90^\circ < \theta < 180^\circ$

5. Let $\vec{u} = 2\vec{i} + \vec{j} - \vec{k}$ and $w = (1, -1, 1)$ give the answer to the following problems. (6 points)

(a) The projection of $\vec{u}$ onto $\vec{w}$ is



$$\overline{W} \cdot \overline{U} = 2 - 1 - 1 = 0$$

$$\overline{W} \perp \overline{U}$$

(b) The distance the point determined by $\vec{w}$ is from the line determined by $\vec{u}$ is

$$\overline{W} = \langle 1, -1, 1 \rangle = \overline{\imath} - \overline{\jmath} + \overline{K}$$

$$\|\overline{W}\| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$$

6. Give the equation of the plane containing the points (1,1,3), (-1,0,4) and (1,2,6). (6 points)

$$P_1 = (1,1,3), \quad P_2 = (-1,0,4), \quad P_3 = (1,2,6)$$

$$\overline{U} = \overrightarrow{P_1P_2} = \langle -2, -1, 1 \rangle$$

$$\overline{V} = \overrightarrow{P_1P_3} = \langle 0, 1, 3 \rangle$$

$$\overline{n} = \overline{U} \times \overline{V} = \begin{vmatrix} \overline{\imath} & \overline{\jmath} & \overline{K} \\ -2 & -1 & 1 \\ 0 & 1 & 3 \end{vmatrix}$$

$$= \overline{\imath}\,(-3-1) - \overline{\jmath}\,(-6-0) + \overline{K}\,(-2-0)$$

$$= -4\overline{\imath} + 6\overline{\jmath} - 2\overline{K}$$

$$(-4\overline{\imath} + 6\overline{\jmath} - 2\overline{K}) \cdot \langle X-1, Y-1, Z-3 \rangle = 0$$

$$-4X + 4 + 6Y - 6 - 2Z + 6 = 0$$

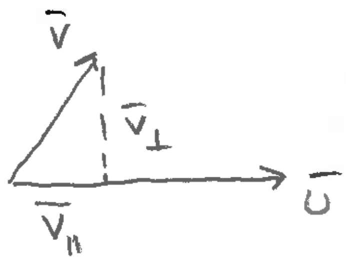
$$-4X + 6Y - 2Z = -4$$

$$2X - 3Y + Z = 2$$

6. Give the area of the parallelogram determined by the projection of $\langle 2,0,-1\rangle$ onto $2\vec{i}-3\vec{k}$ and the normal to the plane $3x-2y+z=0$. (6 points)

$$\bar{v} = \langle 2,0,-1\rangle$$

$$\bar{u} = 2\bar{i} - 3\bar{k}$$



$$\bar{v}_\parallel = \frac{\bar{u}\cdot\bar{v}}{\bar{u}\cdot\bar{u}}\,\bar{u} = \frac{7}{13}(2\bar{i}-3\bar{k}) = \frac{14}{13}\bar{i} - \frac{21}{13}\bar{k}$$

$$\bar{n} = \langle 3,-2,1\rangle$$

$$\bar{v}_\parallel \times \bar{n} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{14}{13} & 0 & -\frac{21}{13} \\ 3 & -2 & 1 \end{vmatrix} = \bar{i}\left(\frac{-42}{13}\right) - \bar{j}\left(\frac{77}{13}\right) + \bar{k}\left(\frac{-28}{13}\right)$$

$$\text{Area} = \|\bar{v}_\parallel \times \bar{n}\| = \frac{1}{13}\sqrt{42^2+77^2+28^2}$$

7. Give the parametric equation of the line that defines where the two planes $x+y+2z=0$ and $2x+4y-z=1$ intersect. (6 points)

$$\bar{n}_1 = \langle 1,1,2\rangle$$

$$\bar{n}_2 = \langle 2,4,-1\rangle$$

$$\bar{v} = \bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 1 & 2 \\ 2 & 4 & -1 \end{vmatrix}$$

$$= \bar{i}(-7) - \bar{j}(-5) + \bar{k}(2)$$

$$= -7\bar{i} + 5\bar{j} + 2\bar{k}$$

$x = 0$

$$y + 2z = 0$$
$$2(4y - z = 1)$$

$$9y = 2,\ y = \frac{9}{2}$$

$$z = -\frac{1}{2}y = -\frac{9}{4}$$

$$P = \left(0, \frac{9}{2}, -\frac{9}{4}\right)$$

$$\ell(t) = \left(0, \frac{9}{2}, \frac{-9}{4}\right) + t\langle -7, 5, 2\rangle$$

$$x = -7t,\ y = \frac{9}{2} + 5t,\ z = \frac{-9}{4} + 2t$$