MATH 237 Sochacki Home Work 2
Oct. 18, 2016 Take Home 50 Points

Name KEY WEST
(Print) (1 point)

All necessary work must be shown for credit. Your work must represent the question asked. You may NOT use computers. You may use your notes or text. Your work must be neat or I will not grade your work. You may discuss this assignment with others, but all work turned in must be your own work and MUST only be on these sheets.

I have neither received nor given help on this exam. Don Key
(Signature) (1 point)

1. Let $\bar{r}(t) = \frac{2t-1}{1+t}\bar{i} + \frac{2\sin 2t}{t}\bar{j} + (1+2t)^{\frac{1}{t}}\bar{k}$. Give the following. (6 points)

(a) The domain for this curve.

$t \neq -1,\ t \neq 0,\ 1+2t > 0 \Rightarrow t > -\frac{1}{2} \Rightarrow (-\frac{1}{2}, 0) \cup (0, \infty)$

(b) $\lim_{t \to 0} \bar{r}(t)$

$\lim_{t \to 0} \frac{2t-1}{1+t} = -1,\quad \lim_{t \to 0} \frac{2\sin 2t}{t} = \lim_{t \to 0} \frac{4\cos 2t}{1} = 4,\quad \lim_{t \to 0} (1+2t)^{\frac{1}{t}}$

$Y = \ln(1+2t)^{\frac{1}{t}} = \frac{\ln(1+2t)}{t}$

$\lim_{t \to 0} Y = \lim_{t \to 0} \frac{\frac{2}{1+2t}}{1} = 2$

$\lim_{t \to 0} (1+2t)^{\frac{1}{t}} = e^2$

$\lim_{t \to 0} \bar{r}(t) = -\bar{i} + 4\bar{j} + e^2\bar{k}$

(c) $\left\|\bar{r}\left(\frac{1}{2}\right)\right\|$

$\bar{r}(\frac{1}{2}) = 0\bar{i} + 4\sin 1\,\bar{j} + 4\bar{k} \quad \|\bar{r}(\frac{1}{2})\| = \sqrt{16\sin^2 1 + 16} = 4\sqrt{1+\sin^2 1}$

2. Consider the curve $\bar{r}(t) = \langle (2-t)e^t, (8-t^2)e^{-t} \rangle$. (12 points)

(a) Give an ACCURATE sketch of this curve for $-\infty < t < \infty$.

4

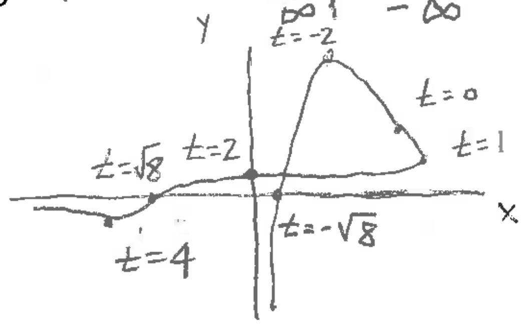
$\bar{r}'(t) = \langle -e^t + (2-t)e^t, -2te^{-t} - (8-t^2)e^{-t} \rangle$

$= \langle (1-t)e^t, (t^2-2t-8)e^{-t} \rangle$

$x' = (1-t)e^t = 0$ $x'+$ $x'-$ 0 1 2

$y' = (t-4)(t+2)e^{-t} = 0$ $y\uparrow$ $y'+$ $y\downarrow$ $y'-$ -2 0 1 2 3 4 5

t	x	y
$-\infty$	0	$-\infty$
$-\sqrt{8}$	$(2+\sqrt{8})e^{-\sqrt{8}}$	0
-2	$4e^{-2}$	$4e^2$
0	2	8
1	e	$7e^{-1}$
$\sqrt{8}$	$(2-\sqrt{8})e^{\sqrt{8}}$	0
4	$-2e^4$	$-4e^{-4}$
∞	$-\infty$	0



$\|\bar{r}'(t)\| = \sqrt{(1-t)^2 e^{2t} + (t-4)^2(t+2)^2 e^{-2t}}$

(b) Give the speed along this curve at $t = 2$.

2

$\left.\frac{ds}{dt}\right|_{t=2} = \|\bar{r}'(2)\| = \sqrt{e^4 + 64e^{-4}}$

(c) Give the unit tangent vector at $t = 2$.

(3) $$\overline{T}(2) = \frac{r'(2)}{\|\bar{r}'(2)\|} = \frac{-e^2 \bar{i} - 8e^{-2}\bar{j}}{\sqrt{e^4+64e^{-4}}}$$

(d) Give the curvature at $t = 2$.

$$\bar{r}'(t) = \langle (1-t)e^t, (t^2-2t-8)e^{-t} \rangle \qquad \kappa = \frac{\|\bar{r}'(2)\times\bar{r}''(2)\|}{\|\bar{r}'(2)\|^3}$$

(3) $$\bar{r}''(t) = \langle -e^t + (1-t)e^t, (2t-2)e^{-t} - (t^2-2t-8)e^{-t} \rangle \qquad = \frac{\left\| \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -e^2 & -8e^{-2} & 0 \\ -2e^2 & 10e^{-2} & 0 \end{vmatrix} \right\|}{(e^4+64e^{-4})^{3/2}}$$

$$\bar{r}'(2) = \langle -e^2, -8e^{-2} \rangle \quad \bar{r}''(2) = \langle -2e^2, 10e^{-2} \rangle$$

$$= \frac{26}{(e^4+64e^{-4})^{3/2}}$$

3. Consider the curve $\bar{r}(t) = \langle 8\cos t + 8t\sin t, 3t^2, 8\sin t - 8t\cos t \rangle$. (12 points)

(a) Give $\overline{T}(s)$.

$$\bar{r}'(t) = \langle -8\sin t + 8\sin t + 8t\cos t, 6t, 8\cos t - 8\cos t + 8t\sin t \rangle$$

$$= \langle 8t\cos t, 6t, 8t\sin t \rangle$$

$$\|\bar{r}'(t)\| = \sqrt{64t^2 + 36t^2} = 10t = \frac{ds}{dt} \Rightarrow s = 5t^2 \Rightarrow t^2 = \frac{s}{5} \Rightarrow t = \sqrt{\frac{s}{5}}$$

(4) $$\bar{r}(s) = \left\langle 8\cos\sqrt{\tfrac{s}{5}} + 8\sqrt{\tfrac{s}{5}}\sin\sqrt{\tfrac{s}{5}}, \tfrac{3}{5}s, 8\sin\sqrt{\tfrac{s}{5}} - 8\sqrt{\tfrac{s}{5}}\cos\sqrt{\tfrac{s}{5}} \right\rangle$$

$$\bar{r}'(s) = \left\langle -\tfrac{4}{5}\left(\tfrac{s}{5}\right)^{-\frac{1}{2}}\sin\sqrt{\tfrac{s}{5}} + \tfrac{4}{5}\left(\tfrac{s}{5}\right)^{-\frac{1}{2}}\sin\sqrt{\tfrac{s}{5}} + 8\sqrt{\tfrac{s}{5}}\,\tfrac{1}{2}\left(\tfrac{s}{5}\right)^{-\frac{1}{2}}\tfrac{1}{5}\cos\sqrt{\tfrac{s}{5}}, \tfrac{3}{5},\right.$$
$$\left. \tfrac{4}{5}\left(\tfrac{s}{5}\right)^{-\frac{1}{2}}\cos\sqrt{\tfrac{s}{5}} - \tfrac{4}{5}\left(\tfrac{s}{5}\right)^{-\frac{1}{2}}\cos\sqrt{\tfrac{s}{5}} + 8\sqrt{\tfrac{s}{5}}\,\tfrac{1}{2}\left(\tfrac{s}{5}\right)^{-\frac{1}{2}}\tfrac{1}{5}\sin\sqrt{\tfrac{s}{5}} \right\rangle$$

$$= \left\langle \tfrac{4}{5}\cos\sqrt{\tfrac{s}{5}}, \tfrac{3}{5}, \tfrac{4}{5}\sin\sqrt{\tfrac{s}{5}} \right\rangle \quad \text{(UNIT vector)}$$

(b) Give $\overline{T}(t), \overline{N}(t), \overline{B}(t)$ at $t = \frac{\pi}{4}$.

$$\overline{T}(t) = \frac{\bar{r}'(t)}{\|\bar{r}'(t)\|} = \left\langle \tfrac{4}{5}\cos t, \tfrac{3}{5}, \tfrac{4}{5}\sin t \right\rangle$$

(3) $$\overline{N}(t) = \frac{\overline{T}'(t)}{\|\overline{T}'(t)\|} = \frac{\langle -\frac{4}{5}\sin t, 0, \frac{4}{5}\cos t \rangle}{\frac{4}{5}} = \langle -\sin t, 0, \cos t \rangle$$

$$\overline{B}\left(\tfrac{\pi}{4}\right) = \overline{T}\left(\tfrac{\pi}{4}\right) \times \overline{N}\left(\tfrac{\pi}{4}\right) = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{2\sqrt{2}}{5} & \frac{3}{5} & \frac{2\sqrt{2}}{5} \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{vmatrix} = \bar{i}\left(\tfrac{3\sqrt{2}}{10}\right) - \bar{j}\left(\tfrac{4}{5}\right) + \bar{k}\left(\tfrac{3\sqrt{2}}{10}\right)$$

(c) Give the curvature at $t=\pi$.

$\bar{r}'(t) = \langle 8t\cos t, 6t, 8t\sin t \rangle$

$\bar{r}''(t) = \langle 8\cos t - 8t\sin t, 6, 8\sin t + 8t\cos t \rangle$

3 $\|\bar{r}'(t)\| = 10t$

$\bar{r}'(\pi) = \langle -8\pi, 6\pi, 0 \rangle$

$\bar{r}''(\pi) = \langle -8, 6, -8\pi \rangle$

$$\mathcal{K} = \frac{\left\| \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -8\pi & 6\pi & 0 \\ -8 & 6 & -8\pi \end{vmatrix} \right\|}{(10\pi)^3} = \frac{\|\bar{i}(-48\pi^2) - \bar{j}(64\pi^2)\|}{1000\pi^3} = \frac{\sqrt{(48)^2+(64)^2}}{1000\pi} = \frac{2}{25\pi}$$

(d) Give the equation of the plane that contains the acceleration vector at $t=\frac{\pi}{4}$.

$\bar{n} = \bar{B}\left(\frac{\pi}{4}\right) = \frac{3\sqrt{2}}{10}\bar{i} - \frac{4}{5}\bar{j} - \frac{3\sqrt{2}}{10}\bar{k}$

2 $P = \bar{r}\left(\frac{\pi}{4}\right) = \langle (4+\pi)\sqrt{2}, \frac{3}{16}\pi^2, (4-\pi)\sqrt{2} \rangle$

$\langle \frac{3\sqrt{2}}{10}, \frac{-4}{5}, \frac{-3\sqrt{2}}{10} \rangle \cdot \langle x-(4+\pi)\sqrt{2}, y - \frac{3}{16}\pi^2, z-(4-\pi)\sqrt{2} \rangle = 0$

$\frac{3\sqrt{2}}{10}x - \frac{4}{5}y - \frac{3\sqrt{2}}{10}z = \frac{3\sqrt{2}}{10}(\pi+4)\sqrt{2} - \frac{3}{20}\pi - \frac{3\sqrt{2}}{10}(4-\pi)\sqrt{2}$

4. Let $z = f(x,y) = 4x+3y$. Describe the graph of this function. (6 points)

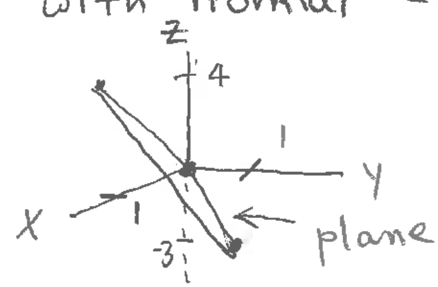
(a) Describe the graph of this function.

$z = 4x+3y$

2 $4x+3y-z=0$ plane

$\bar{n} = \langle 4,3,-1 \rangle$

graph is a plane with normal $\langle 4,3,-1 \rangle$



(b) Simplify $f(x+\Delta x, y+\Delta y) - f(x,y)$.

$f(x+\Delta x, y+\Delta y) = 4(x+\Delta x)+3(y+\Delta y) = 4x+4\Delta x+3y+3\Delta y$

2 $f(x,y) = 4x+3y$

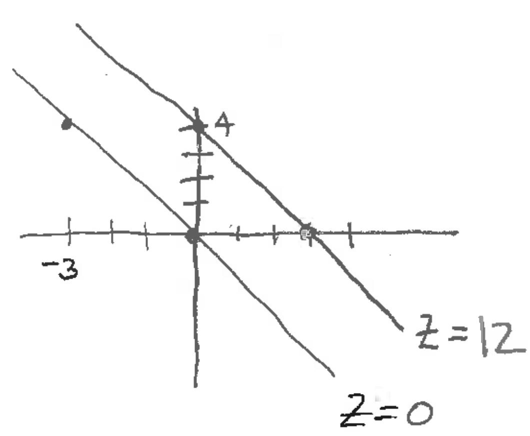
$f(x+\Delta x, y+\Delta y) - f(x,y) = 4\Delta x + 3\Delta y$

(c) Describe the level curves for this function.

$z = c$

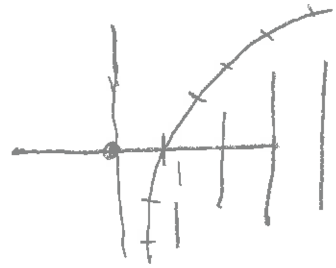
2 $4x+3y = c$ parallel lines

$4x+3y=12$



5. $z = f(x,y) = \dfrac{4}{\sqrt{x^2 - e^y}}$ (8 points)

$x^2 = e^y$



(a) Give the domain and range of f.

$x^2 - e^y = 0$

$e^y = x^2$

$y = \ln x^2 = 2\ln x$

2

$D(f)\quad x^2 > e^y$

$R(f) = (0, \infty)$

(b) Give z_x and z_y.

2

$z = 4(x^2 - e^y)^{-\frac{1}{2}}$

$z_x = -2(x^2 - e^y)^{-3/2}\, 2x = -4x(x^2 - e^y)^{-\frac{3}{2}}$

$z_y = -2(x^2 - e^y)^{-\frac{3}{2}}(-e^y) = 2e^y(x^2 - e^y)^{-\frac{3}{2}}$

(c) Give z_{xy} and z_{yx}.

2

$(z_x)_y = 6x(x^2 - e^y)^{-\frac{5}{2}}(-e^y) = -6xe^y(x^2 - e^y)^{-\frac{5}{2}}$

$(z_y)_x = -3e^y(x^2 - e^y)^{-\frac{5}{2}}(2x) = -6xe^y(x^2 - e^y)^{-\frac{5}{2}}$

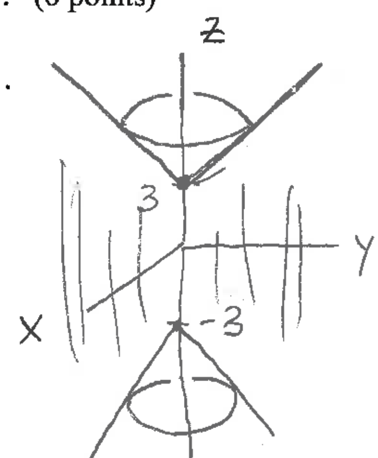
(d) Give z_{xx}

2

$(z_x)_x = -4(x^2 - e^y)^{-\frac{3}{2}} + 6x(x^2 - e^y)^{-\frac{5}{2}}(2x)$

6. Let $w = f(x,y,z) = \sqrt{x^2 + y^2 - z^2 + 9}$. (6 points)

(a) Describe the domain and range of f.



2

$D(f)$

$x^2 + y^2 - z^2 + 9 \geq 0$

$x^2 + y^2 + 9 \geq z^2$

$R(f) = [0, \infty) = w$

(b) Give $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial z}$.

2

$\dfrac{\partial f}{\partial x} = \dfrac{1}{2}(x^2 + y^2 - z^2 + 9)^{-\frac{1}{2}}(2x) = \dfrac{x}{\sqrt{x^2 + y^2 - z^2 + 9}}$

$\dfrac{\partial f}{\partial z} = \dfrac{1}{2}(x^2 + y^2 - z^2 + 9)^{-\frac{1}{2}}(-2z) = \dfrac{-z}{\sqrt{x^2 + y^2 - z^2 + 9}}$

(c) Give $\dfrac{\partial^2 f}{\partial x \partial y}$.

2

$\dfrac{\partial}{\partial y}\left(\dfrac{\partial f}{\partial x}\right) = \dfrac{\partial}{\partial y}\, x(x^2 + y^2 - z^2 + 9)^{-\frac{1}{2}} = \dfrac{-1}{2}x(x^2 + y^2 - z^2 + 9)^{-\frac{3}{2}}(2y)$

$= \dfrac{-xy}{(x^2 + y^2 - z^2 + 9)^{3/2}}$