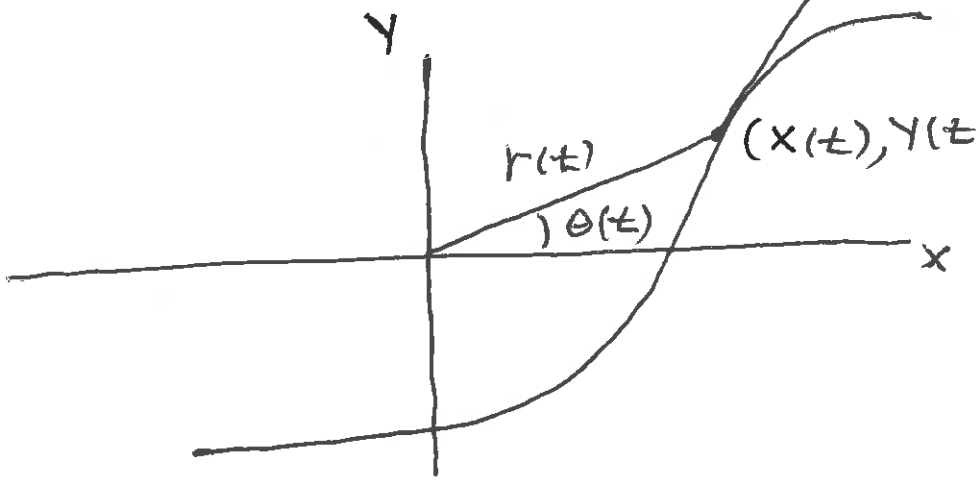
I.



$m = \frac{dy}{dx} = \frac{Y'(t)}{X'(t)}$.

$(X(t), Y(t)) \to (r(t), \theta(t))$

$(r(t)\cos(\theta(t)), r(t)\sin(\theta(t))$

$$\begin{cases} X(t) = r(t)\cos(\theta(t)) \\ X'(t) = r'(t)\cos(\theta(t)) - \theta'(t)r(t)\sin(\theta(t)) \\ X''(t) = r''(t)\cos(\theta(t)) - \theta''(t)r(t)\sin\theta(t) - 2\theta'(t)r'(t)\sin\theta(t) - r(t)(\theta'(t))^2\cos(\theta(t)) \end{cases}$$

$$\begin{cases} Y(t) = r(t)\sin(\theta(t)) \\ Y'(t) = r'(t)\sin(\theta(t)) + \theta'(t)r(t)\cos(\theta(t)) \\ Y''(t) = r''(t)\sin(\theta(t)) + \theta''(t)r(t)\cos(\theta(t)) + 2\theta'(t)r'(t)\cos\theta(t) - r(t)(\theta'(t))^2\sin(\theta(t)) \end{cases}$$

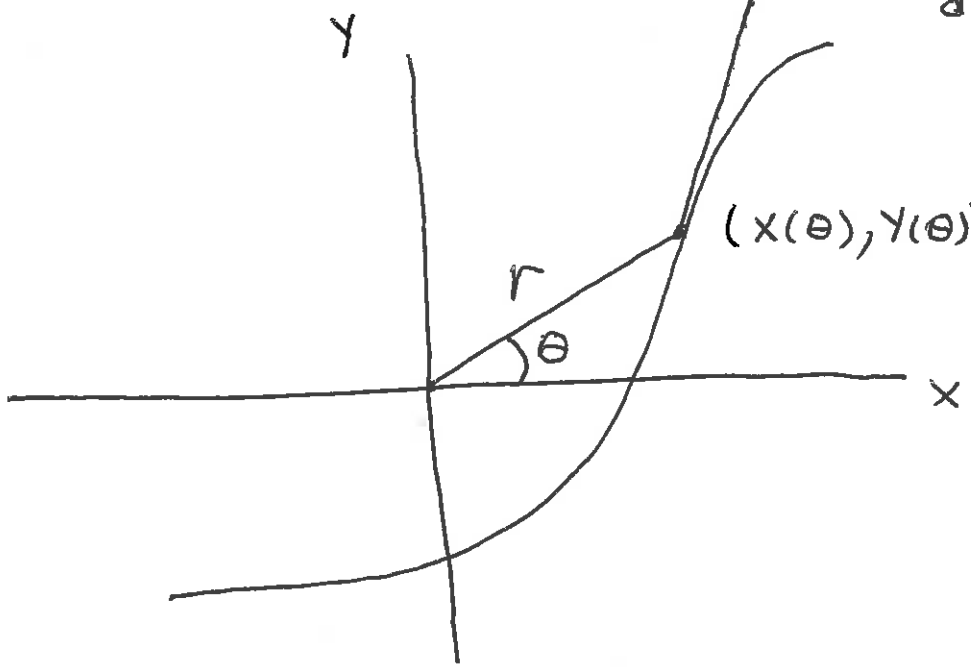
$$\begin{cases} r(t)^2 = X(t)^2 + Y(t)^2 \\ 2r(t)r'(t) = 2X(t)X'(t) + 2Y(t)Y'(t) \\ r'(t) = \frac{X(t)X'(t) + Y(t)Y'(t)}{r(t)} \end{cases}$$

$$\tan(\theta(t)) = \frac{Y(t)}{X(t)}$$

$$\sec^2(\theta(t))\theta'(t) = \frac{Y'(t)X(t) - Y(t)X'(t)}{X(t)^2}$$

$$\theta'(t) = \cos^2(\theta(t))\frac{Y'(t)X(t) - Y(t)X'(t)}{X(t)^2}$$

II.

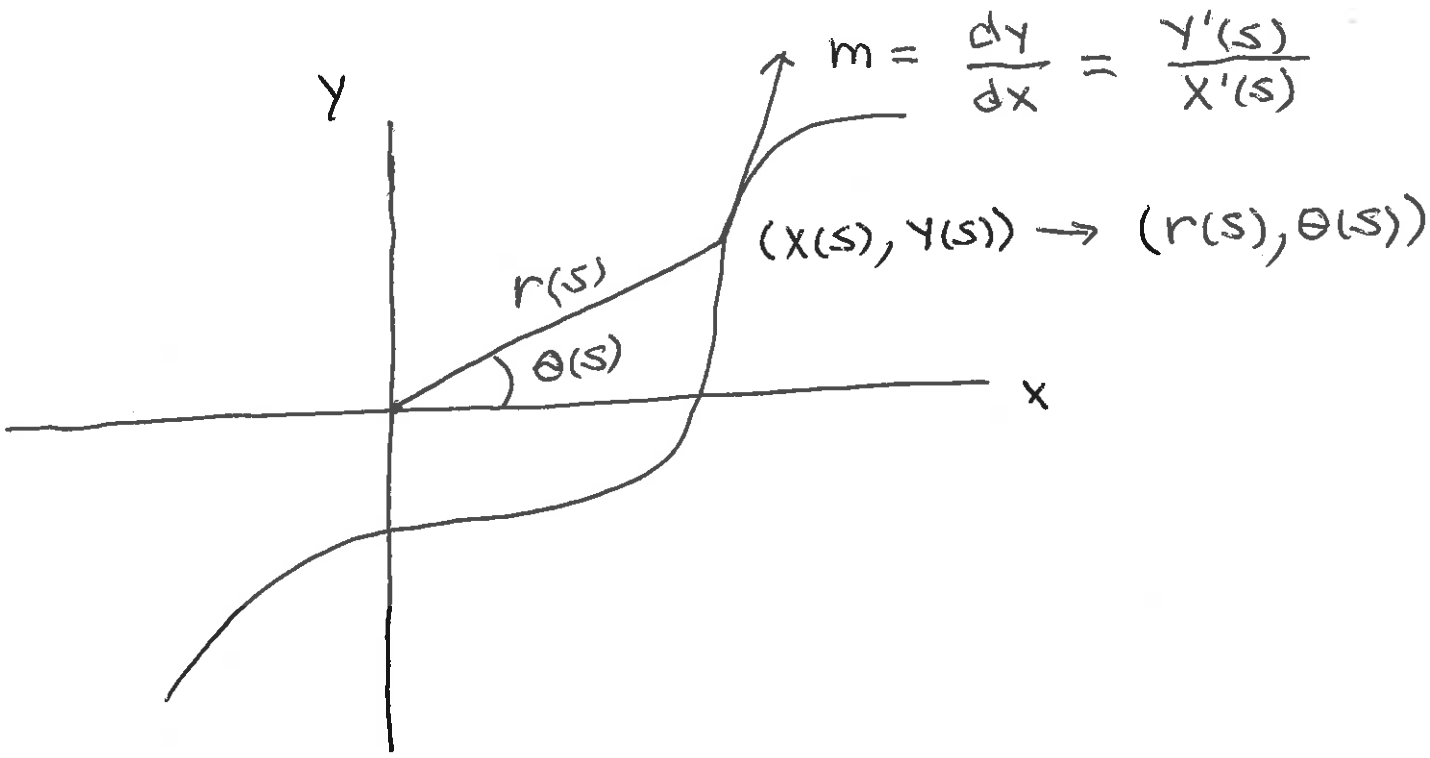


$$m = \frac{dy}{dx} = \frac{Y'(\theta)}{X'(\theta)}$$

$$\begin{cases} X(\theta) = r(\theta)\cos(\theta) \\ X'(\theta) = r'(\theta)\cos\theta - r(\theta)\sin\theta \\ X''(\theta) = r''(\theta)\cos\theta - 2r'(\theta)\sin\theta - r(\theta)\cos\theta \end{cases}$$

$$\begin{cases} Y(\theta) = r(\theta)\sin(\theta) \\ Y'(\theta) = r'(\theta)\sin\theta + r(\theta)\cos\theta \\ Y''(\theta) = r''(\theta)\sin\theta + 2r'(\theta)\cos\theta - r(\theta)\sin\theta \end{cases}$$

III.



$$X(s) = r(s) \cos \theta(s)$$

$$Y(s) = r(s) \sin \theta(s)$$

HW

1. $Y'(x) = \frac{dy}{dx} = \frac{Y'(t)}{X'(t)}$. Give $Y''(x) = \frac{d}{dx}\left(\frac{dy}{dx}\right)$.

2. $Y'(x) = \frac{dy}{dx} = \frac{Y'(\theta)}{X'(\theta)}$. Give $Y''(x) = \frac{d}{dx}\left(\frac{dy}{dx}\right)$.

3. Give $\frac{dy}{dx} = \frac{Y'(\theta)}{X'(\theta)}$ for the single pendulum.