MATH 237 EXAM 1 REVIEW SHEET SEPTEMBER 27, 2016

MATERIAL : 9.1-9.4,10.1-10.6

TOPICS: Parametric Curves, Polar Coordinates, Polar Curves, Arc Length, Polar Areas, Tangents, 3D,Vectors, Dot Products, Cross Products, Lines, Planes

1. Plot the following curves in polar coordinates and give the equation of the tangent line and the length of the tangent vector at θ_0. (a) $\boldsymbol{r} = 2\cos(3\theta)$; $\theta_0 = \frac{\pi}{6}$

   (b) $\boldsymbol{r} = 4 - 2\sin(\theta)$; $\theta_0 = \frac{\pi}{6}$ (c) $\boldsymbol{r} = 3 - 2\cos(2\theta)$; $\theta_0 = \frac{-\pi}{4}$
2. Give the following areas and the arc length integral.
   (a) The area inside $\boldsymbol{r} = \cos(6\theta)$ and the arc length for one loop of the curve.
   (b) The area inside $\boldsymbol{r} = 1 - 2\sin(\theta)$ and the arc length of the inner loop.
   (c) The area inside $\boldsymbol{r} = 2\sin\theta$ and $\boldsymbol{r} = 1 - \sin\theta$.
   (d) The area outside $\boldsymbol{r} = \cos\theta$ but inside $\boldsymbol{r} = \frac{1}{2}$.
3. Plot the following curves, give the equation of the tangent line in vector form at the given t and the arc length.
   (a) $x = 3t, y = 2t^{\frac{3}{2}}$, $t_0 = 1$, arc length for $3 \le t \le 8$. Check your work by writing this curve as $y = f(x)$.
   (b) $x = 2t^{12}; y = 3t^{8}$, $t_0 = 1$, arc length for $0 \le t \le 1$.
   (c) $\left(1 - te^{-t}, 2te^{-t}\right)$, $t_0 = -1$, arc length for $1 \le t \le 2$. Check your work by writing this curve as $y = f(x)$.
4. Give the equation of the sphere whose center is (2, -1, 3) that has a radius vector given by $2\boldsymbol{i} - 3\boldsymbol{k}$.
5. Let $\overline{\boldsymbol{u}} = (1,-2,-3)$ and $\overline{\boldsymbol{v}} = -2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$. Give $\overline{\boldsymbol{v}}_{\parallel}$ and $\overline{\boldsymbol{v}}_{\perp}$.
6. Let $\overline{\boldsymbol{v}} = (1,-3)$ and $\overline{\boldsymbol{w}} = 3\boldsymbol{i} + 2\boldsymbol{j} - 4\boldsymbol{k}$. Give the angle between $\overline{\boldsymbol{v}}$ and $\overline{\boldsymbol{w}}$ and the area of the triangle determined by $\overline{\boldsymbol{v}}$ and $\overline{\boldsymbol{w}}$.
7. Give the equation of the line passing through the two points (1,-2,3) and (2,4,-1).
8. Where does the line passing through (-1,0,1) and (0,2,-1) intersect the plane with normal given by $\overline{\boldsymbol{n}} = 2\boldsymbol{i} - 3\boldsymbol{j}$ and containing the point (1,1,1).
9. Give the equation of the plane passing through (1,-1,2) and whose normal is the cross product of the unit vector determined by the intersection of $2\boldsymbol{x} - 4\boldsymbol{y} + \boldsymbol{z} = 1$ and $\boldsymbol{x} + \boldsymbol{z} = 1$ and $2\boldsymbol{i} - 3\boldsymbol{k}$.
10. Give the equation of the plane normal to the line $(3\boldsymbol{t} - 1)\boldsymbol{i} - (1 + 6\boldsymbol{t})\boldsymbol{j} + 4\boldsymbol{tk}$ passing through the point (-1,2,-1).
11. Give a unit vector parallel to the vector that is normal to the plane $2\boldsymbol{x} - 3\boldsymbol{y} + \boldsymbol{z} = 4$.
12. Give the angle between the line $\frac{\boldsymbol{x} - 1}{2} = \frac{\boldsymbol{y}}{4} = \boldsymbol{z}$ and the plane $2\boldsymbol{x} - 3\boldsymbol{y} + \boldsymbol{z} = 4$.