MATH 237 EXAM 2 REVIEW SHEET OCTOBER 25, 2016

MATERIAL : 11.1-11.5,12.1-12.5

TOPICS: Curves ($\boldsymbol{r}:\Re \to \Re^{\boldsymbol{n}}$) and functions ($f:\mathbb{R}^n \to \mathbb{R}$) ($n=1,2,3$).

1. Graph the following curves. Give the tangent line at the given parameter. Graph the tangent line on your graph. (a) $\mathbf{r}(t)=2\cos(t)\mathbf{i}-3\sin(t)\mathbf{j}$; $t_0=\frac{\pi}{3}$ (b) $\mathbf{r}(t)=2t\mathbf{i}+\ln(2t)\mathbf{j}$; $t_0=1$

   (c) $\mathbf{r}(t)=\cos(t)\mathbf{i}+\sin(t)\mathbf{j}+t^2\mathbf{k}$; $t_0=\frac{5\pi}{3}$ (d) $\mathbf{r}(t)=t^2\mathbf{i}-3t\mathbf{j}+5\mathbf{k}$; $t_0=-2$
2. Give the velocity vector, acceleration vector, and the curvature and arc-length parameter for the following curves. Also, give the tangential and normal components of the acceleration.

(a) $\mathbf{r}(t)=\sin(t^2)\mathbf{i}+\cos(t^2)\mathbf{j}$ (b) $\mathbf{r}(t)=e^t(\cos(t)\mathbf{i}+\sin(t)\mathbf{j})$ (c) $\mathbf{r}(t)=\sin(4t)\mathbf{i}+3t\mathbf{j}+\cos(4t)\mathbf{k}$

(d) $\mathbf{r}(t)=t^2\mathbf{i}+\ln(t)\mathbf{j}+2t\mathbf{k}$

3. Given the following velocities and initial positions give the position vector.

(a) $\mathbf{v}(t)=9t^2\mathbf{i}+e^{-t}\mathbf{j}+2t\mathbf{k}$; $\mathbf{r}(0)=3\mathbf{i}-4\mathbf{j}+\mathbf{k}$ (b) $\mathbf{v}(t)=\cos(4t)\mathbf{i}-3t\mathbf{j}-\sin(4t)\mathbf{k}$; $\mathbf{r}(0)=\mathbf{i}-4\mathbf{k}$

4. Given the following accelerations and initial data give the position vector.

(a) $\mathbf{a}(t)=\sqrt{1+t}\mathbf{i}+e^{3t}\mathbf{j}-2t\mathbf{k}$; $\mathbf{v}(0)=3\mathbf{i}-4\mathbf{j}+\mathbf{k}$; $\mathbf{r}(0)=\mathbf{j}-\mathbf{k}$

(b) $\mathbf{a}(t)=(4-t^2)^2\mathbf{i}-5t^4\mathbf{j}+(1-2t)\mathbf{k}$; $\mathbf{v}(0)=\mathbf{k}$; $\mathbf{r}(0)=\mathbf{i}+\mathbf{k}$

5. Give the normal plane for the following curves at the following parameter value.

(a) $\mathbf{r}(t)=9\cos(4t)\mathbf{i}+9\sin(4t)\mathbf{j}$; $t_0=\frac{7\pi}{4}$ (b) $\mathbf{r}(t)=3t\mathbf{i}+\ln(9t)\mathbf{j}$; $t_0=\frac{1}{9}$ (c) $\mathbf{r}(t)=t^2\mathbf{i}-3t\mathbf{j}+5\mathbf{k}$; $t_0=-2$

6. Give the domain and ranges of the following functions. Also give 5 level curves or level surfaces and describe them. (a) $f(x,y)=\sqrt{4-x^2-9y^2}$ (b) $f(x,y)=e^{4x^2-y}$ (c) $f(x,y)=\ln(y-4x+3)$

(d) $f(x,y,z)=z-\sqrt{x^2+y^2}$ (e) $f(x,y,z)=\ln(2z+3y-4x)$ (f) $f(x,y,z)=e^{\sqrt{100-x^2-4y^2-z^2}}$

7. Let $z=f(x,y)=x^2y^3-\sin(x+y^2)$ and $w=g(x,y,z)=y\sin(z)e^{x^2-y}$. Give the following.

(a) f_x (b) $g_y(0,1,\frac{\pi}{2})$ (c) $\frac{\partial^2 z}{\partial x^2}$ (d) $\frac{\partial w}{\partial x}\big|_{(x,y,z)=(0,1,-\frac{\pi}{2})}$ (e) z_{xy} (f) $\frac{\partial^2 f}{\partial y\partial x}$ (g) $\frac{\partial^3 w}{\partial x^2\partial z}$ (h) g_{xzx}

8. Let $z=e^{\cos(xy^2)}$. Give the directional derivative of z in the $\mathbf{u}=2\mathbf{i}-3\mathbf{j}$ direction. Give the direction of maximum increase in z at $(1,\sqrt{\pi})$.

9. Give the gradients of the following functions.

(a) $f(x,y)=xe^{xy-x^2}$ (b) $w=xyz^3\cos(x^2z)$

10. A hiker is walking along the path given by $\mathbf{r}(t)=t\mathbf{i}+t\sin t\mathbf{j}$. The hiker's altitude is given by $z=x^2-xy+y^2$. Give (a) the lowest point on the hiker's trail. (b) the rate of change in the hiker's altitude at $t=\frac{\pi}{2}$. (c) the direction the hiker should walk in at $t=\frac{5\pi}{2}$ to gain altitude the quickest. (d) the direction the hiker should walk in at $t=6\pi$ to make the walking the easiest.