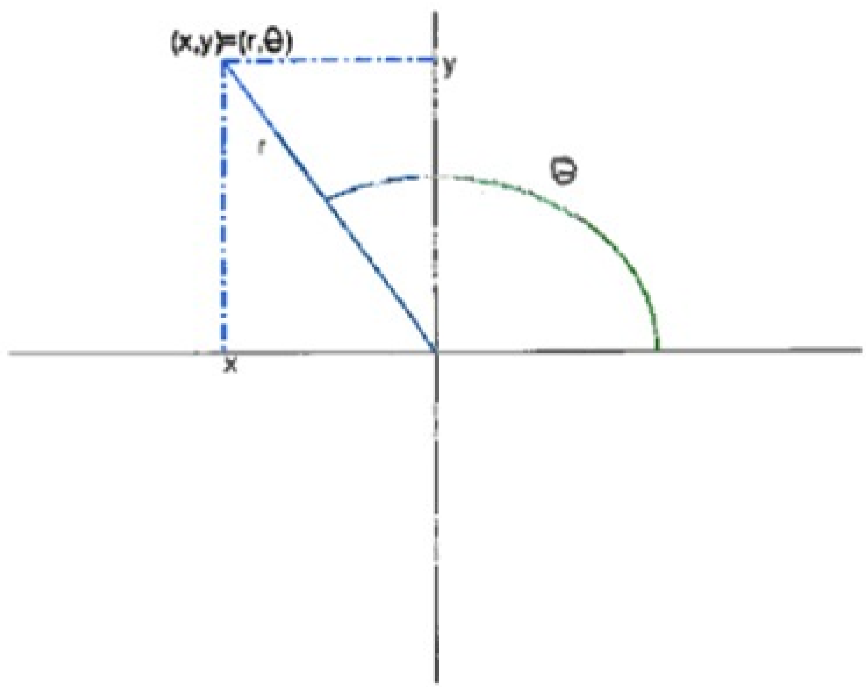
Some Rectangular Coordinate and Polar Coordinate Calculus Relations

We will mostly use (x, y) for rectangular coordinates and (r, θ) for polar coordinates as shown in the figure below. From the figure and what you know about trigonometry we have that

$$x = r\cos\theta \ \text{ and } \ y = r\sin\theta \qquad \text{Equation (1)}$$



Exercise 1. Determine relationships for r and θ in terms of (x, y).

Now suppose r and θ are functions of a variable t, which is sometimes called a parameter. If this is true then as t changes x and y will change and you can plot the points $(x(t), y(t))$ for any t in the domain of r and θ.

Exercise 2. Using the product rule and the chain rule in t determine $x'(t)$ and $y'(t)$ in terms of $r(t), r'(t)$ and/or $\theta(t), \theta'(t)$.
Exercise 3. Sketch a graph of $r(t) = 1$ and $\theta(t) = t$. Give the equation of this graph in rectangular coordinates. Give $x'(t)$ and $y'(t)$ for this graph.

Suppose $x(t) = t$ and $y(t) = f(t)$ then a relationship between x and y is $y = f(x)$. Why? Do you know these curves?

Exercise 4. Sketch the curve $x(t) = t$ and $y(t) = \sqrt{t}$. What is $x'(t)$ and $y'(t)$ for this curve? Are you surprised? What is the equation of this curve in polar coordinates?

There are many ideas we will study that involve $x'(t), y'(t), x''(t), y''(t)$. Why do you think this is the case?

Exercise 5. Using the product rule and the chain rule in t determine $x''(t)$ and $y''(t)$ in terms of $r(t), r'(t), r''(t)$ and/or $\theta(t), \theta'(t), \theta''(t)$.

Exercise 6. Simplify $\sqrt{x'(t)^2 + y'(t)^2}$ in terms of polar coordinates.

Exercise 7. Simplify $\int \sqrt{x'(t)^2 + y'(t)^2}\ dt$ for $x(t) = t$ and $y(t) = 2t^{\frac{3}{2}}$ as far as possible.