MATH 237 Sochacki Team Work 3 25 Points
Nov. 16, 2016

All necessary work must be shown for credit. Your write up should be in pencil. You may NOT use computers, notes or texts. You can use your calculators. All of your team members must help. You cannot have more than 4 team members. Each team member's printed name must be on the left and the written name must be on the right. Your signature means you only worked with team members on this problem. All of the team members below are to work on the following problem together. Each team is to turn in only ONE write up on one side of each of the two sheets given.

1. KEY WEST 1. Mon Key

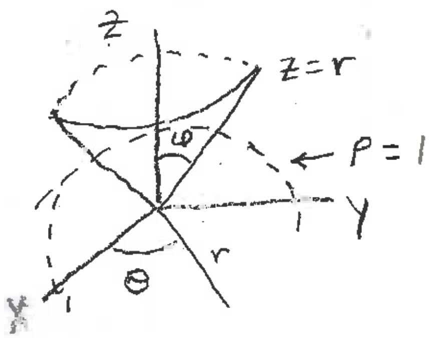
2. ______ 2. ______

3. ______ 3. ______

4. ______ 4. ______

Do one of the following two problems.

1. You are working for E-Mathematics Tutor, an educational software development company. They are asking your team to build a model for their Vector Calculus Tutor division. They want you to set up a rectangular coordinate system, a polar coordinate system and a spherical coordinate system for the volume of the solid inside $z \geq 0, z \geq r, \rho \leq 1$. They want you to evaluate two of these integrals. Your display should be professional.



$z = \sqrt{x^2+y^2}$ $\quad x^2+y^2+z^2=1$

$z = \sqrt{1-x^2-y^2}$

$= \sqrt{1-r^2}$

$\sqrt{x^2+y^2} = \sqrt{1-x^2-y^2}$

$x^2+y^2 = 1-x^2-y^2$

$2x^2+2y^2 = 1$

$x^2+y^2 = \frac{1}{2}$ $\quad r^2 = \frac{1}{2}$

$y^2 = \frac{1}{2} - x^2$ $\quad r = \sqrt{\frac{1}{2}}$

$y = \pm\sqrt{\frac{1}{2}-x^2}$

$$V = \int_{-\sqrt{\frac{1}{2}}}^{\sqrt{\frac{1}{2}}} \int_{-\sqrt{\frac{1}{2}-x^2}}^{\sqrt{\frac{1}{2}-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}} dz\,dy\,dx$$

$$V = \int_0^{2\pi} \int_0^{\sqrt{\frac{1}{2}}} \int_r^{\sqrt{1-r^2}} dz\, r\,dr\,d\theta$$

$$= \int_0^{2\pi} \int_0^{\sqrt{\frac{1}{2}}} z \Big|_r^{\sqrt{1-r^2}} r\,dr\,d\theta$$

$$= \int_0^{2\pi} \int_0^{\sqrt{\frac{1}{2}}} (\sqrt{1-r^2} - r)\, r\,dr\,d\theta$$

$$= \int_0^{2\pi} \left[(1-r^2)^{\frac{1}{2}} r - r^2\right] dr\,d\theta$$

$$= \int_0^{2\pi} -\frac{1}{2}\,\frac{2}{3}(1-r^2)^{\frac{3}{2}} - \frac{r^3}{3} \Big|_0^{\sqrt{\frac{1}{2}}} d\theta$$

$$= \int_0^{2\pi} \frac{-1}{3}(1-r^2)^{\frac{3}{2}} - \frac{r^{3}}{3} \Big|_0^{\sqrt{\frac{1}{2}}} d\theta$$

$$= 2\pi \left(\frac{-1}{3}\left(\frac{1}{2}\right)^{\frac{3}{2}} - \frac{1}{3}\left(\frac{1}{2}\right)^{\frac{3}{2}} + \frac{1}{3} \right)$$

$$= 2\pi \left(\frac{1}{3} - \frac{2}{3}\left(\frac{1}{2}\right)^{\frac{3}{2}} \right)$$

$$V = \int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{1} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$\theta \quad \varphi \quad \rho$

$$= \int_0^{2\pi} \int_0^{\frac{\pi}{4}} \frac{\rho^3}{3} \sin\varphi \Big|_0^1 d\varphi \, d\theta$$

$$= \int_0^{2\pi} \int_0^{\frac{\pi}{4}} \frac{1}{3} \sin\varphi \, d\varphi \, d\theta$$

$$= \int_0^{2\pi} -\frac{1}{3} \cos\varphi \Big|_0^{\frac{\pi}{4}} d\theta$$

$$= 2\pi \left(\frac{-1}{3} \frac{\sqrt{2}}{2} + \frac{1}{3} \right)$$

$$\frac{2}{3}\left(\frac{1}{2}\right)^{\frac{3}{2}} = \frac{2}{3} \frac{1}{2\sqrt{2}} = \frac{1}{3} \frac{\sqrt{2}}{2}$$