MATH 238 Sochacki Assignment 1 Name KEY WEST
Due Mar. 1, 2021 50 Points (Print) (1 point)

All necessary work must be shown for credit and must represent the question asked. Your work MUST be NEAT and your own. You may NOT use computers, notes or texts to look up answers. You may discuss the home work with others in this class, but you must do your own write up. Calculators and computers are only allowed for arithmetic calculations. You must turn the assignment in at the mailbox next to my office door at Roop Hall 115.

I have neither received nor given help on this exam.

Mon Key
(Signature) (1 points)

1. Solve the corresponding homogeneous and non-homogeneous system of linear equations

$$2x_1 + x_2 - 3x_3 - x_4 = 6$$
$$x_1 - 2x_2 + x_3 + 4x_4 = -4$$
$$3x_1 - 2x_2 + 3x_3 + x_4 = 3$$

Give the general answer as a sum of the homogeneous and non-homogeneous solution. (8 points)

$$\left(\begin{array}{cccc|c} 1 & -2 & 1 & 4 & -4 \\ 2 & 1 & -3 & -1 & 6 \\ 3 & -2 & 3 & 1 & 3 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 1 & 4 & -4 \\ 0 & 5 & -5 & -9 & 14 \\ 0 & 4 & 0 & -11 & 15 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 1 & 4 & -4 \\ 0 & 1 & -1 & -\frac{9}{5} & \frac{14}{5} \\ 0 & 4 & 0 & -11 & 15 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 1 & 4 & -4 \\ 0 & 1 & -1 & -\frac{9}{5} & \frac{14}{5} \\ 0 & 0 & 4 & -\frac{19}{5} & \frac{19}{5} \end{array}\right)$$

$$x_1 - 2x_2 + x_3 + 4x_4 = -4$$
$$x_2 - x_3 - \frac{9}{5}x_4 = \frac{14}{5}$$
$$4x_3 - \frac{19}{5}x_4 = \frac{19}{5}$$

$$20x_3 = 19x_4 + 19$$
$$x_2 = x_3 + \frac{9}{5}x_4 + \frac{14}{5}$$
$$x_1 = 2x_2 - x_3 - 4x_4 - 4$$

$$x_4 = t$$
$$x_3 = \frac{19t + 19}{20}$$
$$x_2 = \frac{19t+19}{20} + \frac{9}{5}t + \frac{14}{5} = \frac{55}{20}t + \frac{75}{20} = \frac{11}{4}t + \frac{15}{4}$$
$$x_1 = \frac{11}{2}t + \frac{15}{2} - \frac{19}{20}t - \frac{19}{20} - 4t - 4 = \frac{11}{20}t + \frac{51}{20}$$

$$\bar{x} = \underbrace{t\begin{pmatrix} \frac{11}{20} \\ \frac{11}{4} \\ \frac{19}{20} \\ 1 \end{pmatrix}}_{H} + \underbrace{\begin{pmatrix} \frac{51}{20} \\ \frac{15}{4} \\ \frac{19}{20} \\ 0 \end{pmatrix}}_{NH}$$

$$t = -1 \Rightarrow \bar{x} = \begin{pmatrix} 2 \\ 1 \\ 0 \\ -1 \end{pmatrix} \checkmark$$

2. Determine the vector $\bar{b}$ so that the system of linear equations $A\bar{x} = \bar{b}$ has solutions and indicate how many solutions it has for $A = \begin{pmatrix} 7 & 8 & 9 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$. (10 points)

$$\left(\begin{array}{ccc|c} 7 & 8 & 9 & b_1 \\ 1 & 2 & 3 & b_2 \\ 4 & 5 & 6 & b_3 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & b_2 \\ 7 & 8 & 9 & b_1 \\ 4 & 5 & 6 & b_3 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & b_2 \\ 0 & -6 & -12 & b_1-7b_2 \\ 0 & -3 & -6 & b_3-4b_2 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & b_2 \\ 0 & -6 & -12 & b_1-7b_2 \\ 0 & 0 & 0 & b_3-4b_2-\frac{1}{2}(b_1-7b_2) \end{array}\right)$$

Solns only if $\quad b_3-4b_2-\frac{1}{2}(b_1-7b_2)=0$

$$2b_3 - b_2 - b_1 = 0$$

and then inf many solns

$$\bar{b} = \begin{pmatrix} b_1 \\ b_2 \\ \frac{b_1+b_2}{2} \end{pmatrix}$$

$c-4b-\frac{1}{2}(a-7b)$

$c-\frac{1}{2}b-\frac{1}{2}a$

3. Let $A = \begin{pmatrix} 2 & -4 & 0 & 3 \\ -1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 3 & -6 & -2 & 2 \end{pmatrix}$. Give the determinant of A and the inverse of A if it exists. Give the row reduction matrices that you use. (10 points)

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \left(\begin{array}{cccc|cccc} 2 & -4 & 0 & 3 & 1 & 0 & 0 & 0 \\ -1 & 2 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 3 & -6 & -2 & 2 & 0 & 0 & 0 & 1 \end{array}\right) \text{RS}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \left(\begin{array}{cccc|cccc} -1 & 2 & 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & -4 & 0 & 3 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 3 & -6 & -2 & 2 & 0 & 0 & 0 & 1 \end{array}\right) \text{RS}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{pmatrix} \left(\begin{array}{cccc|cccc} -1 & 2 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 2 & -4 & 0 & 3 & 1 & 0 & 0 & 0 \\ 3 & -6 & -2 & 2 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \left(\begin{array}{cccc|cccc} -1 & 2 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 3 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 0 & 1 \end{array}\right) \text{RS}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{pmatrix} \left(\begin{array}{cccc|cccc} -1 & 2 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 0 & 1 \\ 0 & 0 & 2 & 3 & 1 & 2 & 0 & 0 \end{array}\right)$$

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \left(\begin{array}{cccc|cccc} -1 & 2 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 & -4 & 0 & -2 \end{array}\right)$$

$$\det(A) = (-1)(1)(1)(-1)(-1)^3 = -1$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \left(\begin{array}{cccc|cccc} 1 & -2 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 4 & 0 & 2 \end{array}\right)$$

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \left(\begin{array}{cccc|cccc} 1 & -2 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -4 & 1 & -2 \\ 0 & 0 & 1 & 0 & 2 & -5 & 0 & -3 \\ 0 & 0 & 0 & 1 & -1 & 4 & 0 & 2 \end{array}\right) \to \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 4 & -14 & 2 & -7 \\ 0 & 1 & 0 & 0 & 1 & -4 & 1 & -2 \\ 0 & 0 & 1 & 0 & 2 & -5 & 0 & -3 \\ 0 & 0 & 0 & 1 & -1 & 4 & 0 & 2 \end{array}\right)$$

4. Determine the value of a so that the determinant of the matrix $B = \begin{pmatrix} 3 & a & -3 \\ 1 & 0 & 2 \\ 1 & 1 & 5 \end{pmatrix}$ is 0. Now solve $B\bar{x} = \begin{pmatrix} 6 \\ -1 \\ -4 \end{pmatrix}$ with this value for a. If there is a solution write it as a sum of the homogeneous and non-homogeneous solution. (10 points).

$$\begin{pmatrix} 3 & a & -3 \\ 1 & 0 & 2 \\ 1 & 1 & 5 \end{pmatrix} \xrightarrow{RS} \begin{pmatrix} 1 & 0 & 2 \\ 3 & a & -3 \\ 1 & 1 & 5 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 2 \\ 0 & a & -9 \\ 0 & 1 & 3 \end{pmatrix}$$

$$\det B = -1 \begin{vmatrix} a & -9 \\ 1 & 3 \end{vmatrix} = -(3a+9) = 0 \Rightarrow a = -3$$

$$B = \begin{pmatrix} 3 & -3 & -3 \\ 1 & 0 & 2 \\ 1 & 1 & 5 \end{pmatrix}$$

$$\tilde{B} = \left(\begin{array}{ccc|c} 3 & -3 & -3 & 6 \\ 1 & 0 & 2 & -1 \\ 1 & 1 & 5 & -4 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 3 & -3 & -3 & 6 \\ 1 & 1 & 5 & -4 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & -3 & -9 & 9 \\ 0 & 1 & 3 & -3 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 1 & 3 & -3 \\ 0 & -3 & -9 & 9 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 1 & 3 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$X + 2Z = -1$$
$$Y + 3Z = -3$$

$$Z = t$$
$$Y = -3 - 3t$$
$$X = -1 - 2t$$

$$\bar{X} = \begin{pmatrix} -1-2t \\ -3-3t \\ t \end{pmatrix} = \underbrace{t\begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}}_{H} + \underbrace{\begin{pmatrix} -1 \\ -3 \\ 0 \end{pmatrix}}_{NH}$$

5. Show which of the 8 rules for a vector space the set M_2 with vector addition defined by

(I) If $A, B \in M_2$ then $A \oplus B = \begin{pmatrix} a_{1,1}b_{1,1} & a_{1,2}b_{1,2} \\ a_{2,1}b_{2,1} & a_{2,2}b_{2,2} \end{pmatrix}$

and scalar multiplication defined by

(II) If $A \in M_2$ and $c \in \Re$ then $c \odot A = \begin{pmatrix} ca_{1,1} & a_{1,2} \\ a_{2,1} & ca_{2,2} \end{pmatrix}$.

does or does not satisfy. Explain why M_2 with these two operations does or does not give us a vector space. (10 points)

1) $A \oplus B = \begin{pmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{pmatrix} = \begin{pmatrix} b_{11}a_{11} & b_{12}a_{12} \\ b_{21}a_{21} & b_{22}a_{22} \end{pmatrix} = B \oplus A$ ✓

2) $(A \oplus B) \oplus C = \begin{pmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{pmatrix} \oplus \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11}c_{11} & a_{12}b_{12}c_{12} \\ a_{21}b_{21}c_{21} & a_{22}b_{22}c_{22} \end{pmatrix}$

$A \oplus (B \oplus C) = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \oplus \begin{pmatrix} b_{11}c_{11} & b_{12}c_{12} \\ b_{21}c_{21} & b_{22}c_{22} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11}c_{11} & a_{12}b_{12}c_{12} \\ a_{21}b_{21}c_{21} & a_{22}b_{22}c_{22} \end{pmatrix}$ ✓

3) $A \oplus \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = A \quad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \oplus A = A$ ✓

4) There is NO inverse for $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ OR $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ since $\frac{1}{0}$ D.N.E.

$\left\{ \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \oplus \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{4} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right\}$

5) $c \odot (A \oplus B) = c \odot \begin{pmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{pmatrix} = \begin{pmatrix} ca_{11}b_{12} & a_{12}b_{12} \\ a_{21}b_{21} & ca_{22}b_{22} \end{pmatrix}$

$c \odot A \oplus c \odot B = \begin{pmatrix} ca_{11} & a_{12} \\ a_{21} & ca_{22} \end{pmatrix} \oplus \begin{pmatrix} cb_{11} & b_{12} \\ b_{21} & cb_{22} \end{pmatrix} = \begin{pmatrix} c^2a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & c^2a_{22}b_{22} \end{pmatrix}$

NO

6) $(c_1+c_2) \odot A = \begin{pmatrix} (c_1+c_2)a_{11} & a_{12} \\ a_{21} & (c_1+c_2)a_{21} \end{pmatrix}$

$c_1 \odot A \oplus c_2 \odot A = \begin{pmatrix} c_1a_{11} & a_{12} \\ a_{21} & c_1a_{21} \end{pmatrix} \oplus \begin{pmatrix} c_2a_{11} & a_{12} \\ a_{21} & c_2a_{22} \end{pmatrix}$ NO

$= \begin{pmatrix} c_1c_2a_{11}^2 & a_{12}^2 \\ a_{21}^2 & c_1c_2a_{21}a_{22} \end{pmatrix}$

7) $(c_1 c_2) \odot A = \begin{pmatrix} c_1 c_2 a_{11} & a_{12} \\ a_{21} & c_1 c_2 a_{22} \end{pmatrix}$

$c_1 \odot (c_2 \odot A) = c_1 \odot \begin{pmatrix} c_2 a_{11} & a_{12} \\ a_{21} & c_2 a_{22} \end{pmatrix}$

$= \begin{pmatrix} c_1 c_2 a_{11} & a_{12} \\ a_{21} & c_1 c_2 a_{22} \end{pmatrix}$ ✓

8) $1 \odot A = \begin{pmatrix} 1 a_{11} & a_{12} \\ a_{21} & 1 a_{22} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = A$ ✓

NOT a V.S.