MATH 238 Sochacki Assignment 2
Due Mar. 26, 2021 50 Points

Name KEY WEST
(Print) (1 point)

All necessary work must be shown for credit and must represent the question asked. Your work MUST be NEAT and your own and done in PENCIL. You may NOT use computers, notes or texts to look up answers. You may discuss the home work with others in this class, but you must do your own write up. Calculators and computers are only allowed for arithmetic calculations. You must turn the assignment in at the mailbox next to my office door at Roop Hall 115.

I have neither received nor given help on this exam.

Tim Key
(Signature) (1 point)

1. **Show** whether or not the following are vector spaces. (6 points each)

(a) The $A \epsilon M_n$ that are invertible.

$A = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$ are inv

$A + B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ is NOT inv

Not closed under vector addition

Not a v.s.

(b) The polynomials satisfying $p(0) = p'(1)$.

Let $r(0) = r'(1)$ and $g(0) = g'(1)$

then

$$r(0) + g(0) = r'(1) + g'(1)$$
$$(r+g)(0) = (r+g)'(1)$$
closed under vector addition

$$c\,r(0) = c\,r'(1)$$
$$(cr)(0) = (cr)'(1)$$
closed under scalar mult.

Is a v.s.

2. **Show** whether or not $1 + x + x^2 + x^3 \epsilon$ span$\{2x^3, 1 - x + 2x^2, 3x - x^2\}$. (6 points)

$c_1 2x^3 + c_2(1 - x + 2x^2) + c_3(3x - x^2) = 1 + x + x^2 + x^3$

$2c_1 = 1 \quad c_2 = 1 \quad -c_2 + 3c_3 = 1 \quad 2c_2 - c_3 = 1$

$c_1 = \frac{1}{2}$

$-c_2 + 3c_3 = 1$
$2c_2 - c_3 = 1$

$-c_2 + 3c_3 = 1$
$5c_3 = 2 \quad c_3 = \frac{2}{5}$

$c_2 = 3c_3 - 1$
$= 3\left(\frac{2}{5}\right) - 1 = \frac{1}{5}$

IMPOSSIBLE
NO soln
NOT in span

3. **Show** whether or not $B = \left\{\begin{pmatrix} 1 & -1 \\ -2 & 2 \end{pmatrix}, \begin{pmatrix} -2 & 2 \\ 1 & -1 \end{pmatrix}, \begin{pmatrix} 1 & -2 \\ -1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & -2 \\ -4 & 5 \end{pmatrix}\right\}$ is a basis for M_2. (6 points)

$c_1\begin{pmatrix} 1 & -1 \\ -2 & 2 \end{pmatrix} + c_2\begin{pmatrix} -2 & 2 \\ 1 & -1 \end{pmatrix} + c_3\begin{pmatrix} 1 & -2 \\ -1 & 2 \end{pmatrix} + c_4\begin{pmatrix} 1 & -2 \\ -4 & 5 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$c_1 - 2c_2 + c_3 \quad c_4 = 0$
$-c_1 + 2c_2 - 2c_3 - 2c_4 = 0$
$-2c_1 + c_2 - c_3 - 4c_4 = 0$
$2c_1 - c_2 + 2c_3 + 5c_4 = 0$

$$\begin{pmatrix} 1 & -2 & 1 & 1 \\ -1 & 2 & -2 & -2 \\ -2 & 1 & -1 & -4 \\ 2 & -1 & 2 & 5 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -2 & 1 & 1 \\ 0 & 0 & -1 & -1 \\ 0 & -3 & 1 & -2 \\ 0 & 3 & 0 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & 1 & 1 \\ 0 & 0 & -1 & -1 \\ 0 & -3 & 1 & -2 \\ 0 & 0 & 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -2 & 1 & 1 \\ 0 & 0 & -1 & -1 \\ 0 & -3 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

inf many solns to H problem
lin. dep. NOT a basis

4. Give a basis for the NS(A) and the CS(A) and the dimension of each for $A = \begin{pmatrix} 3 & 1 & -3 & -1 \\ 1 & 0 & 2 & 0 \\ 1 & 1 & 5 & 4 \end{pmatrix}$. (12 points).

NS

$$\begin{pmatrix} 3 & 1 & -3 & -1 \\ 1 & 0 & 2 & 0 \\ 1 & 1 & 5 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 2 & 0 \\ 3 & 1 & -3 & -1 \\ 1 & 1 & 5 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -9 & -1 \\ 0 & 1 & 3 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -9 & -1 \\ 0 & 0 & 12 & 5 \end{pmatrix}$$

$12x_3 + 5x_4 = 0$

$x_4 = 12t$

$x_3 = -5t$

$x_2 = 9x_3 + x_4 = -33t$

$x_1 = -2x_3 = 10t$

$NS(A) = \text{span}\left\{ \begin{pmatrix} 10 \\ -33 \\ -5 \\ 12 \end{pmatrix} \right\}$

$\dim(NS(A)) = 1$

$CS(A) = RS(A^T)$

$$\begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 1 \\ -3 & 2 & 5 \\ -1 & 0 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 3 & 1 & 1 \\ -3 & 2 & 5 \\ -1 & 0 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 2 & 8 \\ 0 & 0 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 12 \\ 0 & 0 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 12 \\ 0 & 0 & 0 \end{pmatrix}$$

Basis $= \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 12 \end{pmatrix} \right\}$

$\dim(CS(A)) = 3$

5. Give a phase portrait for the differential equation $y' = 4y - y^3$. Be sure to include the equilibrium solutions, the concavity and enough possible solutions. (12 points)

$$y' = y(4-y^2) = y(2-y)(2+y)$$

EQ: $y(x) = 0,\ y(x) = 2,\ y(x) = -2$

$$y'' = 4y' - 3y^2y' = y'(4-3y^2)$$

$$y'' = 0,\quad y^2 = \frac{4}{3} \Rightarrow y = \pm\frac{2}{\sqrt{3}}$$