MATH 238 Sochacki Assignment 3 Name KEY WEST
Due Apr. 12, 2021 50 Points (Print) (1 point)

All necessary work must be shown for credit and must represent the question asked. Your work MUST be NEAT and your own and done in PENCIL. You may NOT use computers, notes or texts to look up answers. You are to do this homework **totally** on your own. You may ask at the SMLC for help with the concepts. Calculators and computers are only allowed for arithmetic calculations. You must turn the assignment in at the mailbox next to my office door at Roop Hall 115 by 7:00 PM on April 12.

I have neither received nor given help on this assignment.

Mon Key
(Signature) (1 point)

1. You invest $1000 in an investment fund that pays 5% annual interest compounded continuously. After that, you invest $250 a month continuously. How much money do you have after 10 years? (12 points)

$A(t) \rightarrow$ amount of money at time t in years

$A'(t) = 0.05A(t) + 250 \cdot 12 \qquad A(0) = 1000$

$A'(t) - 0.05A(t) = 3000 \qquad u(t) = e^{\int -0.05\,dt} = e^{-0.05t}$

$(Ae^{-0.05t})' = 3000e^{-0.05t}$

$Ae^{-0.05t} = -60{,}000\,e^{-0.05t} + C$

$A = -60{,}000 + Ce^{0.05t}$

$A(0) = -60{,}000 + C = 1000 \rightarrow C = 61{,}000$

$A(t) = -60{,}000 + 61{,}000e^{0.05t}$

$A(10) \approx \$40{,}572$

5 point Bonus problem
Do Problem 1 with no monthly investment for monthly compounding and continuous compounding. Give the difference in the amount of money in these two plans after 25 years.

$A' = 0.05A \qquad A(0) = 1000$

$A = Ce^{0.05t} \qquad A(0) = C = 1000$

$A = 1000e^{0.05t}$

$A(25) \approx 3490.34$

$A_K = A_0\left(1 + \frac{r}{12}\right)^{12K}$

$A_{25} = 1000\left(1 + \frac{0.05}{12}\right)^{12 \cdot 25}$

≈ 3481.29

Close ≈ 9.05

2. A 1000 gallon tank contains 5 pounds of salt. Salt water containing 0.5 pounds of salt per gallon is poured into the tank at 2 gallons per minute. The well stirred tank is drained at 3 gallons per minute. How much salt is in the tank when the tank has 750 gallons in it? (12 points)

$S(t) \rightarrow$ amount of salt in lbs at t minutes

$S(0) = 5$

$S'(t) =$ amount in $-$ amount out

$$= 0.5 \frac{\text{lbs}}{\text{gal}} \cdot 2 \frac{\text{gal}}{\text{min}} - \frac{S(t)}{1000-t} \frac{\text{lbs}}{\text{gal}} \cdot 3 \frac{\text{gal}}{\text{min}}$$

$$S' = 1 - \frac{3}{1000-t} S$$

$$S' + \frac{3}{1000-t} S = 1 \qquad v = e^{\int \frac{3}{1000-t}\, dt}$$

$$= e^{-3\ln(1000-t)} = (1000-t)^{-3}$$

$$\left(S(1000-t)^{-3}\right)' = (1000-t)^{-3}$$

$$S(1000-t)^{-3} = \frac{(1000-t)^{-2}}{2} + C$$

$$S = \frac{1000-t}{2} + C(1000-t)^{3}$$

$$S(0) = 500 + C(1000)^3 = 5$$

$$C = \frac{5-500}{(1000)^3} = -495 \times 10^{-9}$$

$$S(t) = \frac{1000-t}{2} - (495 \times 10^{-9})(1000-t)^3$$

$$S(250) \approx 166.171 \text{ lbs.}$$

3. SHOW whether or not $\{e^{2x}, x, \cos x\}$ forms a linearly independent or dependent set. Be VERY clear. (12 points)

Using the Wronskian

$$\begin{vmatrix} e^{2x} & x & \cos x \\ 2e^{2x} & 1 & -\sin x \\ 4e^{2x} & 0 & -\cos x \end{vmatrix} =$$

$$-x \begin{vmatrix} 2e^{2x} & -\sin x \\ 4e^{2x} & -\cos x \end{vmatrix} + \begin{vmatrix} 2e^{2x} & \cos x \\ 4e^{2x} & -\cos x \end{vmatrix}$$

$$= -x\left(-2e^{2x}\cos x + 4e^{2x}\sin x\right) + \left(-e^{2x}\cos x - 4e^{2x}\cos x\right)$$

$$= 2xe^{2x}\cos x - 4xe^{2x}\sin x - 5e^{2x}\cos x$$

$$= e^{2x}\left((2x-5)\cos x - 4x\sin x\right)$$

$W(0) = -5 \neq 0$ so lin. ind.

4. Consider the initial value ordinary differential equation y'' + 2y' – 3y = 0; y(0) = 1, y'(0) =1. (a) Solve this problem. (b) Write this problem as a matrix system of ODEs and determine the eigenvalues of this system. (c) If u = y', sketch a direction (slope) field for $\frac{du}{dy}$ that shows the behavior of the solution for the entire phase field. (12 points)

(a) $y'' + 2y' - 3y = 0$

$r^2 + 2r - 3 = 0$

$(r+3)(r-1) = 0 \quad r = -3, 1$

$y(t) = c_1 e^{-3t} + c_2 e^{t}$ $\quad y(0) = c_1 + c_2 = 1$

$y' = -3c_1 e^{-3t} + c_2 e^{t}$ $\quad y'(0) = -3c_1 + c_2 = 1$

$4c_1 = 0$

$c_2 = 1$

$y = e^{t}$

(b) Let $u = y'$ $\quad y' = u$

$u' = y'' = 3y - 2y' = 3y - 2u$

$\begin{pmatrix} y \\ u \end{pmatrix}' = \begin{pmatrix} 0 & 1 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} y \\ u \end{pmatrix}$ $\quad \begin{vmatrix} -\lambda & 1 \\ 3 & -2-\lambda \end{vmatrix} = \lambda^2 + 2\lambda - 3$

$\lambda = -3, 1$

(c) $\frac{du}{dy} = \frac{u'}{y'} = \frac{3y - 2u}{u}$ $\quad \frac{du}{dy} = 0 = 3y - 2u$

$u = \frac{3}{2}y$