MATH 238 Sochacki Assignment 4 Name KEY WEST
Due May 7, 2021 50 Points (Print) (1 point)

All necessary work must be shown for credit and must represent the question asked. Your work MUST be NEAT and your own and done in PENCIL. You may NOT use computers, notes or texts to look up answers. You are to do this homework **totally** on your own. You may ask at the SMLC for help with the concepts. Calculators and computers are only allowed for arithmetic calculations. You must turn the assignment in at the mailbox next to my office door at Roop Hall 115 by 12:00 PM (NOON) on Friday May 7.

I have neither received nor given help on this assignment.

Mon Key

(Signature) (1 point)

1. Solve the initial value ordinary differential equation
$4y'' - 12y' + 9y = 0;\ y(1) = 1, y'(1) = 0.$ (6 points)

$4r^2 - 12r + 9 = 0$

$(2r-3)(2r-3) = 0$

$r = 3/2$

$Y_H = c_1 e^{\frac{3}{2}x} + c_2 x e^{\frac{3}{2}x}$

$Y_H' = \frac{3}{2} c_1 e^{\frac{3}{2}x} + c_2\left(e^{\frac{3}{2}x} + \frac{3}{2} x e^{\frac{3}{2}x}\right)$

$Y_H(1) = c_1 e^{\frac{3}{2}} + c_2 e^{\frac{3}{2}} = 1$

$Y_H'(1) = \frac{3}{2} c_1 e^{\frac{3}{2}} + c_2 \frac{5}{2} e^{\frac{3}{2}} = 0$

$-\frac{5}{2}$

$c_1 e^{\frac{3}{2}} = \frac{5}{2}$

$c_1 = \frac{5}{2} e^{-\frac{3}{2}}$ $\quad c_2 = \frac{1 - c_1 e^{\frac{3}{2}}}{e^{\frac{3}{2}}}$

$= -\frac{3}{2} e^{-\frac{3}{2}}$

$Y_H = \frac{5}{2} e^{-\frac{3}{2}} e^{\frac{3}{2}x} - \frac{3}{2} e^{-\frac{3}{2}} x e^{\frac{3}{2}x}$

2. Solve the initial value ordinary differential equation
$3y'' + 6y' + 6y = 0; y(0) = 0, y'(0) = 1.$
Describe the behavior of the solution in terms of the mass-spring system. (6 points)

$3r^2 + 6r + 6 = 0$

$r^2 + 2r + 2 = 0$

$r = \dfrac{-2 \pm \sqrt{4-8}}{2}$

$= -1 \pm i$

$Y_H = c_1 e^{-t} \cos t + c_2 e^{-t} \sin t$

$Y_H(0) = c_1 = 0$

$Y_H = c_2 e^{-t} \sin t$

$Y_H' = c_2 (e^{-t} \cos t - e^{-t} \sin t)$

$Y_H'(0) = c_2 = 1$

$Y_H = e^{-t} \sin t$

The mass oscillates decreasingly to 0.

3. Solve the ordinary differential equation $y''' - 4y' = 16x - \sin 2x + e^{2x}$. (12 points)

$r^3 - 4r = 0$

$r(r^2 - 4) = 0$

$r = 0, \pm 2$

$Y_H = c_1 + c_2 e^{2x} + c_3 e^{-2x}$

$Y = c_1 + c_2 e^{2x} + c_3 e^{-2x}$
$-2x^2 - \frac{1}{16}\cos 2x + \frac{1}{8} x e^{2x}$

$\underline{\underline{Y_N}}$ for $16x$ $\quad Y_N = x(Ax + B) = Ax^2 + Bx$

$Y_N' = 2Ax + B, \quad Y_N'' = 2A, \quad Y_N''' = 0$

$Y_N''' - 4Y_N' = 0 - 4(2Ax + B) = 16x$

$-8A = 16 \quad -4B = 0$

$A = -2 \quad B = 0$

for $-\sin 2x$ $\quad Y_N = A\cos 2x + B\sin 2x$

$Y_N' = -2A\sin 2x + 2B\cos 2x$

$Y_N'' = -4A\cos 2x - 4B\sin 2x$

$Y_N''' = 8A\sin 2x - 8B\cos 2x$

$Y_N''' - 4Y_N' = 16A\sin 2x - 16B\cos 2x = -\sin 2x$

$16A = -1 \quad -16B = 0$

$A = -\frac{1}{16} \quad B = 0$

for e^{2x} $\longrightarrow$ $Y_N = Axe^{2x}$ $\quad Y_N' = A(e^{2x} + 2xe^{2x})$

$= Ae^{2x}(2x+1)$

$Y_N''' - 4Y_N' = Ae^{2x}(8) = e^{2x}$

$8A = 1 \rightarrow A = \frac{1}{8}$

$Y_N'' = A[2e^{2x}(2x+1) + 2e^{2x}]$

$= Ae^{2x}(4x+4)$

$Y_N''' = A[2e^{2x}(4x+4) + 4e^{2x}] = Ae^{2x}(8x+12)$

4. Solve the ordinary differential equation $x^2y'' - 6y = 10 - 25x^3$. Show that $y_1 = x^3$ is a solution to the corresponding homogeneous problem. (12 points)

$$Y_1 = X^3, \quad Y_1' = 3X^2, \quad Y_1'' = 6X \Rightarrow X^2 Y_1'' - 6Y_1 = X^2(6X) - 6X^3 = 0$$

$$Y_2 = UY_1 = UX^3, \quad Y_2' = U'X^3 + 3X^2U, \quad Y_2'' = U''X^3 + 3X^2U' + 3X^2U' + 6XU$$
$$= U''X^3 + 6X^2U' + 6XU$$

$$X^2Y_2'' - 6Y_2 = X^2(U''X^3 + 6X^2U' + 6XU) - 6UX^3$$
$$= X^2(U''X^3 + 6X^2U') = 0 \Rightarrow U''X + 6U' = 0 \qquad V = U'$$
$$V'X + 6V = 0$$
$$\frac{V'}{V} = \frac{-6}{X}$$
$$\ln V = -6 \ln X$$
$$V = X^{-6}$$
$$U = \frac{X^{-5}}{-5}$$

$$Y_2 = UX^3 = X^{-5}X^3 = X^{-2}$$

$$W(X^3, X^{-2}) = \begin{vmatrix} X^3 & X^{-2} \\ 3X^2 & -2X^{-3} \end{vmatrix} = -5$$

$$U_1 = \int \frac{\begin{vmatrix} 0 & X^{-2} \\ \frac{10-25X^3}{X^2} & -2X^{-3} \end{vmatrix}}{-5} dx$$

$$= \int (2X^{-4} - 5X^{-1})dx = \frac{-2}{3}X^{-3} - 5\ln|X|$$

$$U_2 = \int \frac{\begin{vmatrix} X^3 & 0 \\ 3X^2 & \frac{10-25X^3}{X^2} \end{vmatrix}}{-5} dx = \int (-2X + 5X^4)dx$$
$$= -X^2 + X^5$$

$$Y = C_1X^3 + C_2X^{-2} + \left(\frac{-2}{3}X^{-3} - 5\ln|X|\right)X^3 + (-X^2 + X^5)X^{-2}$$
$$= C_1X^3 + C_2X^{-2} - \frac{5}{3} - 5X^3\ln|X|$$

5. Give the eigenvalues and corresponding eigenvectors of A, A^T, A^{-1} for $A = \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ -3 & 2 & 3 \end{pmatrix}$.

(12 points)

$$\det(A-\lambda I) = \begin{vmatrix} -1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 0 \\ -3 & 2 & 3-\lambda \end{vmatrix} = (-1-\lambda)\left((1-\lambda)(3-\lambda)\right) + 3(1-\lambda)$$

$$= (1-\lambda)\left((-1-\lambda)(3-\lambda)+3\right)$$

$$= (1-\lambda)(\lambda^2-2\lambda) = (1-\lambda)(\lambda)(\lambda-2)$$

$$\lambda = 1, 0, 2 \qquad A^{-1} \text{ D.N.E.}$$

$\underline{\lambda=0}$ $\begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ -3 & 2 & 3 \end{pmatrix} \Rightarrow \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 2 & 0 \end{pmatrix}$

$V_2 = 0 \quad -V_1 + V_3 = 0 \quad \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$\underline{\lambda=1}$ $\begin{pmatrix} -2 & 0 & 1 \\ 0 & 0 & 0 \\ -3 & 2 & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} -2 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{pmatrix}$

$V_3 = 2V_1 \quad V_1 = -2V_2 \quad \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix}$

$\underline{\lambda=2}$ $\begin{pmatrix} -3 & 0 & 1 \\ 0 & -1 & 0 \\ -3 & 2 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} -3 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 2 & 0 \end{pmatrix}$

$V_2 = 0 \quad V_3 = 3V_1 \quad \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$

$\underline{A^T}$ $\begin{pmatrix} -1 & 0 & -3 \\ 0 & 1 & 2 \\ 1 & 0 & 3 \end{pmatrix}$

$\underline{\lambda=0}$ $\begin{pmatrix} -1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$

$V_2 = -2V_3 \quad V_1 = -3V_3 \quad \begin{pmatrix} -3 \\ -2 \\ 1 \end{pmatrix}$

$\underline{\lambda=1}$ $\begin{pmatrix} -2 & 0 & -3 \\ 0 & 0 & 2 \\ 1 & 0 & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} -2 & 0 & -3 \\ 0 & 0 & 2 \\ 1 & 0 & 0 \end{pmatrix}$

$V_1 = 0, \; V_3 = 0 \quad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\underline{\lambda=2}$ $\begin{pmatrix} -3 & 0 & -3 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} -3 & 0 & -3 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$

$V_2 = 2V_3 \quad V_1 = -V_3 \quad \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$