Definition of a Vector Space

A set V with two operations $\bigoplus$ and $\bigodot$ called vector addition and scalar multiplication, respectively is a vector space if and only if the following eight properties hold for all elements (vectors) $\bar{v} \in V$ and all scalars.

1. $\bar{v} \bigoplus \bar{u} = \bar{u} \bigoplus \bar{v}$ (commutative property)
2. $(\bar{u} \bigoplus \bar{v}) \bigoplus \bar{w} = \bar{u} \bigoplus (\bar{v} \bigoplus \bar{w})$ (associative property)
3. There is a vector $\bar{\mathbf{0}} \in V$ so that $\bar{v} \bigoplus \bar{\mathbf{0}} = \bar{v}$
4. For each $\bar{v} \in V$ there is a vector $-\bar{v} \in V$ so that $\bar{v} \bigoplus (-\bar{v}) = \bar{\mathbf{0}}$
5. $c \bigodot (\bar{v} \bigoplus \bar{u}) = c \bigodot \bar{v} \bigoplus c \bigodot \bar{u}$ (scalar vector distributive property)
6. $(a+b) \bigodot \bar{v} = a \bigodot \bar{v} \bigoplus b \bigodot \bar{v}$ (scalar distributive property)
7. $(ab) \bigodot \bar{v} = a(b \bigodot \bar{v})$ (scalar associative property)
8. $(1) \bigodot \bar{v} = \bar{v}$ (scalar identity property)

We will usually write $\bigoplus$ as $+$ and $\bigodot$ as $\cdot$