MATH 238 Sochacki Exam 1
Feb. 24, 2021 100 Points

Name KEY WEST
(Print) (2 points)

All necessary work must be shown for credit and must represent the question asked. Your work MUST be NEAT. You may NOT use computers, notes or texts. Calculators can be used only to help with arithmetic.

I have neither received nor given help on this exam. Don Key
(Signature) (2 points)

Does not exist is always a possible answer but NOT necessarily a correct answer.

1. Let $A=\begin{pmatrix} 0 & -2 & 1 \\ 3 & -2 & 4 \\ 0 & 2 & -1 \\ -3 & 2 & -4 \end{pmatrix}$, $B=\begin{pmatrix} 1 & -2 \\ -2 & 1 \\ 1 & 3 \end{pmatrix}$ and $\bar{b}=\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$. Give the following. (6 points each)

(a) $A\bar{b}$

DNE

(b) AB

$$\begin{pmatrix} 0 & -2 & 1 \\ 3 & -2 & 4 \\ 0 & 2 & -1 \\ -3 & 2 & -4 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ -2 & 1 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 5 & 1 \\ 11 & 4 \\ -5 & -1 \\ -11 & -4 \end{pmatrix}$$

(c) BA

DNE

(d) The solutions to $A\bar{x}=\bar{b}$.

$$\left(\begin{array}{ccc|c} 0 & -2 & 1 & 1 \\ 3 & -2 & 4 & 2 \\ 0 & 2 & -1 & 3 \\ -3 & 2 & -4 & 4 \end{array}\right) \qquad \left(\begin{array}{ccc|c} 0 & -2 & 1 & 1 \\ 3 & -2 & 4 & 2 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 6 \end{array}\right) \leftarrow \text{No soln}$$

2. Let $A = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 3 & 2 \\ 0 & 6 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 3 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & \frac{5}{2} & 0 \\ 0 & 0 & 0 \\ 0 & 6 & 4 \end{pmatrix}$. Give (8 points each)

(a) the matrix L so that LA = B

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

(b) the matrix L so that LA = C

$$L = \begin{pmatrix} 1 & 0 & \frac{1}{4} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix}$$

(c) the determinant of L in part (a).

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3. Determine a so that the solution to the system of linear equations $\begin{matrix} 3x+9y+6z=1 \\ x+7y+4z=1 \\ 2x+8y+5z=a \end{matrix}$ has a solution. How many solutions does the system have for this a? (12 points)

$$\left(\begin{array}{ccc|c} 1 & 7 & 4 & 1 \\ 3 & 9 & 6 & 1 \\ 2 & 8 & 5 & a \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 7 & 4 & 1 \\ 0 & -12 & -6 & -2 \\ 0 & -6 & -3 & a-2 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 7 & 4 & 1 \\ 0 & -12 & -6 & -2 \\ 0 & 0 & 0 & a-1 \end{array}\right)$$

$a=1$

$-12y-6z=-2$

inf many solns.

4. Give the determinant and inverse of the matrix $A=\begin{pmatrix} 2 & 4 & -1 \\ 0 & -2 & 4 \\ 1 & 2 & 0 \end{pmatrix}$. (12 points)

$$\left(\begin{array}{ccc|ccc} 2 & 4 & -1 & 1 & 0 & 0 \\ 0 & -2 & 4 & 0 & 1 & 0 \\ 1 & 2 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 0 & 0 & 1 \\ 0 & -2 & 4 & 0 & 1 & 0 \\ 2 & 4 & -1 & 1 & 0 & 0 \end{array}\right) \text{RS}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 0 & 0 & 1 \\ 0 & -2 & 4 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & -2 \end{array}\right)$$

$\det A = (-1)(1)(-2)(-1) = -2$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 0 & 0 & 1 \\ 0 & -1 & 2 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -1 & 0 & 2 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 2 & \frac{1}{2} & -4 \\ 0 & 0 & 1 & -1 & 0 & 2 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & 1 & -7 \\ 0 & 1 & 0 & -2 & -\frac{1}{2} & 4 \\ 0 & 0 & 1 & -1 & 0 & 2 \end{array}\right)$$

$\underbrace{\begin{matrix} 4 & 1 & -7 \\ -2 & -\frac{1}{2} & 4 \\ -1 & 0 & 2 \end{matrix}}_{A^{-1}}$

5. Give the solution to corresponding homogeneous problem for

$$\begin{aligned} 2x_1 - x_2 - 6x_3 &= 8 \\ -x_1 + x_2 + 2x_3 &= -3 \\ 2x_1 - 3x_2 - 2x_3 &= 5 \end{aligned}$$

and write the solution to this problem as a linear combination of solutions to the homogenous problem and the non-homogeneous problem. (24 points)

$$\left(\begin{array}{ccc|c} -1 & 1 & 2 & -3 \\ 2 & -1 & -6 & 8 \\ 2 & -3 & -2 & 5 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -1 & 1 & 2 & -3 \\ 0 & 1 & -2 & 2 \\ 0 & -1 & 2 & -1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -1 & 1 & 2 & -3 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

H SLE

$$x_2 - 2x_3 = 0 \qquad x_2 = 2x_3 = 2t$$

$$-x_1 + x_2 + 2x_3 = 0 \qquad x_1 = x_2 + x_3 = 3t$$

$$\bar{X}_H = \begin{pmatrix} 3t \\ 2t \\ t \end{pmatrix} = t\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

NH SLE has NO soln

$$0x_1 + 0x_2 + 0x_3 \neq 1$$