MATH 238 – Sochacki
Exam 1 – February 22, 2021
Sections 1.1-1.6 SLE, Matrices, Determinants

1. Solve the following SLEs. Indicate how many solutions there are. Write the solutions as a sum of the solution to the corresponding homogeneous solution and a solution to the non-homogeneous problem. Give the matrices L that you used to solve the SLE.

(a) $\begin{aligned} 2x-4y&=10\\ x+2y&=-3 \end{aligned}$ (b) $\begin{aligned} 2x-4y&=1\\ -x+2y&=3 \end{aligned}$ (c) $x+y-z=1$ (d) $\begin{aligned} 2x+4y-3z&=1\\ -x+2y&=1 \end{aligned}$ (e) $\begin{aligned} 2u_1+3u_2-u_3+u_4&=2\\ 3u_1-2u_2+u_3-u_4&=2\\ u_1+u_2-2u_3+2u_4&=1 \end{aligned}$

(f) $\begin{aligned} 3v_1+v_2-v_3+v_4-v_5&=3\\ 6v_1+2v_2-2v_3+v_4-v_5&=6\\ v_1-v_2+v_3-v_4+v_5&=1 \end{aligned}$

2. Determine a, b, c so that the SLE $\begin{aligned} 2x+5y+8z&=a\\ x+4y+7z&=b\\ 3x+6y+9z&=c \end{aligned}$ has none, one or infinitely many solutions. For the case of infinitely many solutions also give the solution as a sum of the corresponding homogeneous solution and non-homogeneous solution.

2. Determine which of the following matrices have inverses. Give the matrices L that produce the inverse.

(a) $A=\begin{pmatrix} 4 & 3\\ -1 & 2 \end{pmatrix}$ (b) $A=\begin{pmatrix} 2 & -4\\ -1 & 2 \end{pmatrix}$ (c) $A=\begin{pmatrix} 4 & -2 & 3\\ 0 & 0 & 9\\ 0 & 0 & 6 \end{pmatrix}$ (d) $A=\begin{pmatrix} 8 & 15 & -21\\ 0 & -20 & 36\\ -16 & -40 & 60 \end{pmatrix}$

3. Give the inverse of the following matrix and give all the L matrices that you used to find the inverse.

$$F=\begin{pmatrix} 2 & 4 & -1 & 2\\ 1 & 2 & -1 & 3\\ -1 & -3 & 0 & 3\\ -3 & -6 & -3 & -1 \end{pmatrix}$$

4. Give the determinant of the following matrices and the determinant of their transposes.

$$A=\begin{pmatrix} 1 & 2 & 3\\ 7 & 8 & 9\\ 4 & 5 & 6 \end{pmatrix}\quad B=\begin{pmatrix} 1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 8 \end{pmatrix}\quad C=\begin{pmatrix} -1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9 \end{pmatrix}\quad F=\begin{pmatrix} 1 & 2 & -1 & 2\\ 2 & 4 & 1 & 3\\ -1 & 2 & 0 & 3\\ -2 & 0 & 2 & -1 \end{pmatrix}$$

5. Put A, B, C and F from problem 4 in upper triangular form and give the determinants of these matrices. Which of these matrices have inverses?

6. Show that the inverse of A^T is $\left(A^T\right)^{-1}$ and $\left(A^{-1}\right)^T$. What can you conclude?

7. What is the inverse of A^2, A^3?