DIFFERENTIAL CALCULUS

Definition of the Limit

$\lim_{x\to c} f(x) = L$ means that:

For all $\epsilon > 0$, there exists a $\delta > 0$ such that if $x \in (c-\delta, c) \cup (c, c+\delta)$, then $f(x) \in (L-\epsilon, L+\epsilon)$.

Equivalently, in terms of absolute value inequalities, $\lim_{x\to c} f(x) = L$ means that:

For all $\epsilon > 0$, there exists a $\delta > 0$ such that if $0 < |x-c| < \delta$, then $|f(x) - L| < \epsilon$.

The Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then there exist values M and m in the interval $[a, b]$ such that $f(M)$ is the maximum value of $f(x)$ on $[a, b]$ and $f(m)$ is the minimum value of $f(x)$ on $[a, b]$.

The Intermediate Value Theorem

If f is continuous on a closed interval $[a, b]$, then for any K strictly between $f(a)$ and $f(b)$, there exists at least one $c \in (a, b)$ such that $f(c) = K$.

Definition of the Derivative

The derivative of a function $f(x)$ is defined to be the function:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

or, equivalently:

$$f'(x) = \lim_{z\to x} \frac{f(z) - f(x)}{z - x}.$$

Derivative Rules

$\frac{d}{dx}(kf(x)) = kf'(x)$

$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$

$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$

$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

$\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$

Rolle's Theorem

If f is continuous on $[a, b]$ and differentiable on (a, b), and if $f(a) = f(b) = 0$, then there exists at least one value $c \in (a, b)$ for which $f'(c) = 0$.

The Mean Value Theorem

If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists at least one value $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Derivatives of Basic Functions

$\frac{d}{dx}(x^k) = kx^{k-1}$

$\frac{d}{dx}(e^{kx}) = ke^{kx}$

If $b > 0$ and $b \neq 1$, then $\frac{d}{dx}(b^x) = (\ln b)b^x$

$\frac{d}{dx}(\ln|x|) = \frac{1}{x}$

If $b > 0$ and $b \neq 1$, then $\frac{d}{dx}(\log_b|x|) = \frac{1}{(\ln b)x}$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}(\sinh x) = \cosh x$

$\frac{d}{dx}\cosh x = \sinh x$

$\frac{d}{dx}\tanh x = \operatorname{sech}^2 x$

INTEGRAL CALCULUS

Definition of the Definite Integral

If f is defined on $[a, b]$ then the definite integral of f on $[a, b]$ is

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\,\Delta x,$$

if this limit exists, where $\Delta x = \frac{b-a}{n}$, $x_k = a + k\,\Delta x$, and x_k^* is any choice of point in $[x_{k-1}, x_k]$.

Definition of the Indefinite Integral

The indefinite integral of a continuous function f is the family of antiderivatives

$$\int f(x)\,dx = F(x) + C,$$

where F is any antiderivative of f.

Fundamental Theorem of Calculus

If f is continuous on $[a, b]$ and F is any antiderivative of F, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Integrals of Basic Functions

$\int x^k\,dx = \frac{1}{k+1}x^{k+1} + C$ (for $k \neq -1$)

$\int \frac{1}{x}\,dx = \ln|x| + C$

$\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C$ (for $k \neq 0$)

$\int b^x\,dx = \frac{1}{\ln b}b^x + C$ (for $b > 0$ and $b \neq 1$)

$\int \ln x\,dx = x\ln x - x + C$

$\int \sin x\,dx = -\cos x + C$

$\int \cos x\,dx = \sin x + C$

$\int \sec^2 x\,dx = \tan x + C$

$\int \csc^2 x\,dx = -\cot x + C$

$\int \sec x \tan x\,dx = \sec x + C$

$\int \csc x \cot x\,dx = -\csc x + C$

$\int \tan x\,dx = -\ln|\cos x| + C$

$\int \cot x\,dx = \ln|\sin x| + C$

$\int \sec x\,dx = \ln|\sec x + \tan x| + C$

$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$

$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$

$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$

$\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1} x + C$

$\int \sin^{-1} x\,dx = x\sin^{-1} x + \sqrt{1-x^2} + C$

$\int \tan^{-1} x\,dx = x\tan^{-1} x - \frac{1}{2}\ln(x^2+1) + C$

$\int \sinh x\,dx = \cosh x + C$

$\int \cosh x\,dx = \sinh x + C$

$\int \operatorname{sech}^2 x\,dx = \tanh x + C$

$\int \frac{1}{\sqrt{x^2+1}}\,dx = \sinh^{-1} x + C$

$\int \frac{1}{\sqrt{x^2-1}}\,dx = \cosh^{-1} x + C$

$\int \frac{1}{1-x^2}\,dx = \tanh^{-1} x + C$