MATH 238 Sochacki Exam 1
May 23, 2014 In Class Part 50 Points

Name KEY WEST
(Print) (1 point)

All necessary work must be shown for credit and must represent the question asked. Your work MUST be NEAT. You may NOT use computers, notes or texts. Calculators can be used only to help with arithmetic.

I have neither received nor given help on this exam. Don Key
(Signature) (1 point)

1. Let $A = \begin{pmatrix} 1 & -2 & 0 \\ 2 & -3 & 5 \\ -1 & 2 & 0 \\ -2 & 3 & -5 \end{pmatrix}$ and $\bar{b} = \begin{pmatrix} -1 \\ 2 \\ -3 \\ 4 \end{pmatrix}$. Write down a corresponding system of linear equations and indicate how many solutions there are to this SLE. (6 points)

$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 2 & -3 & 5 & 2 \\ -1 & 2 & 0 & -3 \\ -2 & 3 & -5 & 4 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 5 & 4 \\ 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 6 \end{array}\right) \Big\} \text{ No Soln.}$$

2. Let $A \in M_n$ be an invertible matrix. Write down 3 statements that are equivalent to this statement. (6 points)

1) A is Gauss-Jordan equivalent to an upper or lower triangular matrix with no zeroes on the diagonal

2) $A\bar{x} = \bar{0}$ has only $\bar{x} = \bar{0}$ as a soln.

3) $A\bar{x} = \bar{b}$ has only $\bar{x} = A^{-1}\bar{b}$ as a soln.

3. Give the inverse of the matrix $A = \begin{pmatrix} 3 & 4 \\ -2 & -1 \end{pmatrix}$. (4 points)

$$\left(\begin{array}{cc|cc} 3 & 4 & 1 & 0 \\ -2 & -1 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{cc|cc} -2 & -1 & 0 & 1 \\ 3 & 4 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{cc|cc} 1 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 3 & 4 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{cc|cc} 1 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & \frac{5}{2} & 1 & \frac{3}{2} \end{array}\right)$$

$$\left(\begin{array}{cc|cc} 1 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{2}{5} & \frac{3}{5} \end{array}\right)$$

$$\begin{pmatrix} 1 & 0 & -\frac{1}{5} & -\frac{4}{5} \\ 0 & 1 & \frac{2}{5} & \frac{3}{5} \end{pmatrix}$$

$$A^{-1} = \frac{1}{5}\begin{pmatrix} -1 & -4 \\ 2 & 3 \end{pmatrix}$$

4. If $A = \begin{pmatrix} 2 & 0 & -3 \\ 1 & 2 & 1 \\ 0 & -3 & 3 \end{pmatrix}$ give A^T and A^2. (4 points)

$$A^T = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & -3 \\ -3 & 1 & 3 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 2 & 0 & -3 \\ 1 & 2 & 1 \\ 0 & -3 & 3 \end{pmatrix}\begin{pmatrix} 2 & 0 & -3 \\ 1 & 2 & 1 \\ 0 & -3 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 9 & -15 \\ 4 & 1 & 2 \\ -3 & -15 & 6 \end{pmatrix}$$

5. If $A^{-1} = \begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix}$ and $B^{-1} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ what is $(AB)^{-1}$? (4 points)

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$= \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix} = \begin{pmatrix} 5 & -5 \\ -1 & 3 \end{pmatrix}$$

6. Give the matrix $E \in M_3$ that when multiplied by $A \in M_3$ switches the last two rows of A and multiplies the first row of A by -2 and puts the answer in the first row of A. Give the inverse of E. (6 points)

$$E = \left(\begin{array}{ccc|ccc} -2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right)$$

$$E^{-1} = \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

7. Let $A \in M_{3X4}, B \in M_{2X3}, C \in M_{3X3}$. Indicate which of AB, BA, AC, CA are defined and give the size of each of the defined matrix products and the sizes of the transposes of the matrix products. (6 points)

AB DNE

$BA \in M_{2\times 4}$

AC DNE

$CA \in M_{3\times 4}$

$(BA)^T \in M_{4\times 2}$

$(CA)^T \in M_{4\times 3}$

8. Give a solution to the system of linear equations $\begin{aligned} 3x - y + 2z &= 3 \\ x + 2y - z &= 4 \\ 2x + 11y - 7z &= 17 \end{aligned}$. Solve the corresponding homogeneous system of linear equations. Give a description of all the solutions to this system of linear equations. (12 points)

$$\left(\begin{array}{ccc|c} 3 & -1 & 2 & 3 \\ 1 & 2 & -1 & 4 \\ 2 & 11 & -7 & 17 \end{array}\right) \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 3 & -1 & 2 & 3 \\ 2 & 11 & -7 & 17 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & -7 & 5 & -9 \\ 0 & 7 & -5 & 9 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & -7 & 5 & -9 \\ 0 & 0 & 0 & 0 \end{array}\right) \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

HP: $\begin{pmatrix} -3 \\ 5 \\ 7 \end{pmatrix}$

NP: $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

Soln: $\bar{x} = c\begin{pmatrix} -3 \\ 5 \\ 7 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 1-3c \\ 2+5c \\ 7+c \end{pmatrix}$