MATH 238 Sochacki Exam 1
May 30, 2014 In Class Part 100 Points

Name KEY WEST
(Print) (2 points)

All necessary work must be shown for credit and must represent the question asked. Your work MUST be NEAT. You may NOT use computers, notes or texts. Calculators can be used only to help with arithmetic.

I have neither received nor given help on this exam. Don Key
(Signature) (2 points)

1. If $A = \begin{pmatrix} 2 & 1 & -3 \\ 1 & -2 & 1 \\ 3 & -6 & 1 \end{pmatrix}$ and $\det(EA) = 5$, what is $\det(E)$? (12 points)

$$\begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & -3 \\ 3 & -6 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & 1 \\ 0 & 5 & -5 \\ 0 & 0 & -2 \end{pmatrix} \quad \det = -10 \quad \det A = 10$$

$$\det(EA) = \det(E)\det(A)$$

$$5 = \frac{1}{2} \cdot 10$$

$$\uparrow$$

2. Let $A \in M_n$ with $\det(A) \neq 0$. Write down 4 statements that are equivalent to this statement. (12 points)

a) A is inv
b) $A\bar{x} = \bar{0}$ has only $\bar{x} = \bar{0}$ as a soln.
c) $A\bar{x} = \bar{b}$ has only $\bar{x} = A^{-1}\bar{b}$ as a soln.
d) A is Gauss-Jordan row equivalent to I.
e) The rows of A are lin. ind.
f) The cols of A are lin. ind.

3. Show whether or not $W = \left\{ \begin{pmatrix} a \\ 1-a \end{pmatrix} \mid a \in \mathbb{R} \right\}$ is a subspace of $\mathbb{R}^2$. (12 points)

$$\begin{pmatrix} a \\ 1-a \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow a = 0 \text{ and } a = 1 \quad \text{impossible}$$

4. Show whether or not $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ is in the span of $\left\{ \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ -1 & -1 \end{pmatrix} \right\}$. (12 points)

$$c_1 \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} + c_2 \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix} + c_3 \begin{pmatrix} 1 & 0 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$\begin{aligned} c_1 + c_3 &= 1 \\ -c_1 - c_2 &= 2 \\ c_2 - c_3 &= 3 \\ 2c_1 + c_2 - c_3 &= 4 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ -1 & -1 & 0 & 2 \\ 0 & -1 & -1 & 3 \\ 2 & 1 & -1 & 4 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & -1 & 1 & 3 \\ 0 & -1 & -1 & 3 \\ 0 & 1 & -3 & 2 \end{array}\right)$$

$$\left.\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & -1 & 1 & 3 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & -2 & 5 \end{array}\right)\right\} \underline{\text{NO}} \text{ SOLN}$$

5. Show whether or not the columns of $A = \begin{pmatrix} 1 & 2 \\ -2 & -4 \end{pmatrix}$ are linearly independent or dependent. (12 points)

$$\begin{pmatrix} 1 & 2 \\ -2 & -4 \end{pmatrix} \to \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}$$

dependent

$$\begin{pmatrix} 1 \\ -2 \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

6. Let $A = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -4 \\ 3 & 6 & 7 \end{pmatrix}$. Give a basis for $NS(A)$. Show whether or not $\bar{b} = \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$ is in $span(CS(A))$.

(12 points)

$$\begin{pmatrix} 1 & -2 & 3 \\ 0 & 0 & 2 \\ 0 & 12 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & 3 \\ 0 & 12 & -2 \\ 0 & 0 & 2 \end{pmatrix} \Rightarrow A \text{ is inv.} \quad NS(A) = \{\bar{0}\}$$

Since A is inv, $\Rightarrow \bar{b} \in CS(A)$.

7. Give a basis for $span\left\{\begin{pmatrix}1\\0\\-1\end{pmatrix},\begin{pmatrix}0\\1\\0\end{pmatrix},\begin{pmatrix}0\\2\\-2\end{pmatrix},\begin{pmatrix}2\\-1\\0\end{pmatrix}\right\}$. (12 points)

$$\begin{pmatrix}1&0&-1\\0&1&0\\0&2&-2\\2&-1&0\end{pmatrix}\rightarrow\begin{pmatrix}1&0&-1\\0&1&0\\0&2&-2\\0&-1&2\end{pmatrix}\rightarrow\begin{pmatrix}1&0&-1\\0&1&0\\0&0&-2\\0&0&2\end{pmatrix}$$

$$\Downarrow$$

basis $\begin{pmatrix}1\\0\\-1\end{pmatrix},\begin{pmatrix}0\\1\\0\end{pmatrix},\begin{pmatrix}0\\0\\2\end{pmatrix}$

8. Show whether or not $B=\{1-x,1+x,4-3x^2\}$ is a basis for P_2. Give the coordinates of $1+4x-x^2$ in terms of B. (12 points)

$$c_1(1-x)+c_2(1+x)+c_3(4-3x^2)=1+4x-x^2$$

$$c_1+c_2+4c_3=1 \quad -c_1+c_2=4 \quad -3c_3=-1$$

$$c_1+c_2=\frac{-1}{3} \qquad c_3=\frac{1}{3}$$

$$-c_1+c_2=4$$

$$2c_2=\frac{-1}{3}+\frac{12}{3}=\frac{11}{3} \qquad c_2=\frac{11}{6}$$

$$c_1=\frac{-1}{3}-c_2=\frac{-1}{3}-\frac{11}{6}=\frac{-13}{6}$$

Since one and only one soln and dim $P_2=3$, this is a basis.