Homogeneous SLE, SLE, Kernel, Null Space, Inverse, Determinant Homework

Each matrix *A* with vectors $\bar{\mathbf{b}}_1, \bar{\mathbf{b}}_2$ below defines a SLE. For each problem below, give *a* so that the SLE has (i) zero solutions (ii) one solution and (iii) infinitely many solutions. (If there is no answer, explain why.) For each problem give a set of vectors that define the kernel or null space of *A*. Express all your answers in the form $\bar{\mathbf{u}}+\bar{\mathbf{v}}$ where $A\bar{\mathbf{u}}=\bar{\mathbf{0}}$, $A\bar{\mathbf{v}}=\bar{\mathbf{b}}$. If the matrix has a determinant, give the determinant. If the matrix has an inverse, give the inverse. Write down all the E matrices you use to determine the inverse.

1. $A=\begin{pmatrix} 2 & -4 \\ 3 & 6 \end{pmatrix}, \bar{\mathbf{b}}_1=\begin{pmatrix} 1 \\ a \end{pmatrix}, \bar{\mathbf{b}}_2=\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
2. $A=\begin{pmatrix} 2 & -4 \\ 3 & 5 \end{pmatrix}, \bar{\mathbf{b}}_1=\begin{pmatrix} a \\ 1 \end{pmatrix}, \bar{\mathbf{b}}_2=\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
3. $A=\begin{pmatrix} 2 & -4 \\ 1 & 3 \\ -6 & 12 \end{pmatrix}, \bar{\mathbf{b}}_1=\begin{pmatrix} 1 \\ 0 \\ a \end{pmatrix}, \bar{\mathbf{b}}_2=\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$
4. $A=\begin{pmatrix} 2 & -1 & 3 \\ 1 & 1 & -2 \\ 3 & -6 & 15 \end{pmatrix}, \bar{\mathbf{b}}_1=\begin{pmatrix} 1 \\ a \\ 1 \end{pmatrix}, \bar{\mathbf{b}}_2=\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$
5. $A=\begin{pmatrix} 2 & -4 & 6 \\ -1 & 2 & -3 \end{pmatrix}, \bar{\mathbf{b}}_1=\begin{pmatrix} 1 \\ a \end{pmatrix}, \bar{\mathbf{b}}_2=\begin{pmatrix} 0 \\ 2 \end{pmatrix}$
6. $A=\begin{pmatrix} 3 & -2 & 1 & 0 \\ 1 & -1 & 2 & 1 \\ 2 & -2 & -3 & -2 \\ -3 & 2 & -1 & 1 \end{pmatrix}, \bar{\mathbf{b}}_1=\begin{pmatrix} 1 \\ 1 \\ a \\ 1 \end{pmatrix}, \bar{\mathbf{b}}_2=\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$
7. $A=\begin{pmatrix} 2 & -3 & 0 & 1 \\ -1 & 3 & 2 & 0 \\ 3 & -3 & 2 & 2 \\ 1 & 3 & 6 & 2 \end{pmatrix}, \bar{\mathbf{b}}_1=\begin{pmatrix} 1 \\ 1 \\ 1 \\ a \end{pmatrix}, \bar{\mathbf{b}}_2=\begin{pmatrix} b \\ -1 \\ 2 \\ 3 \end{pmatrix}$

Let P, A be invertible matrices. Give formulas for $(PAP^{-1})^n$ and $[(PAP^{-1})^T]^n$ for $n \in Z$.