MATH 238 Sochacki Exam 1
May 23, 2014 Take Home Part 25 Points

Name KEY WEST
(Print)

All necessary work must be shown for credit and must represent the question asked. Your work MUST be NEAT. You may NOT use computers, notes or texts. Multiple choice questions must be CLEARLY circled.

I have neither received nor given help on this exam. Don Key
(Signature) (1 points)

1. Let $A=\begin{pmatrix}3&6&2&2\\1&2&1&-3\\4&4&-2&2\\2&3&0&3\end{pmatrix}$. Show if the inverse of A exists. Give all the row reduction matrices E that you use to determine this. (6 points)

$$\left(\begin{array}{cccc|cccc}3&6&2&2&1&0&0&0\\1&2&1&-3&0&1&0&0\\4&4&-2&2&0&0&1&0\\2&3&0&3&0&0&0&1\end{array}\right)$$

$$\downarrow$$

$$\left(\begin{array}{cccc|cccc}1&2&1&-3&0&1&0&0\\3&6&2&2&1&0&0&0\\4&4&-2&2&0&0&1&0\\2&3&0&3&0&0&0&1\end{array}\right)$$

E_1

$$\left(\begin{array}{cccc|cccc}1&2&1&-3&0&1&0&0\\0&0&-1&11&1&-3&0&0\\0&-4&-6&14&0&-4&1&0\\0&-1&-2&9&0&-2&0&1\end{array}\right) \quad E_2=\begin{pmatrix}1&0&0&0\\-3&1&0&0\\-4&0&1&0\\-2&0&0&1\end{pmatrix}$$

$$\left(\begin{array}{cccc|cccc}1&2&1&-3&0&1&0&0\\0&-1&-2&9&0&-2&0&1\\0&-4&-6&14&0&-4&1&0\\0&0&-1&11&1&-3&0&0\end{array}\right) \quad E_3=\begin{pmatrix}1&0&0&0\\0&0&0&1\\0&0&1&0\\0&1&0&0\end{pmatrix}$$

$$\left(\begin{array}{cccc|cccc}1&2&1&-3&0&1&0&0\\0&-1&-2&9&0&-2&0&1\\0&0&2&-22&0&4&1&-4\\0&0&-1&11&1&-3&0&0\end{array}\right) \quad E_4=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&-4&1&0\\0&0&0&1\end{pmatrix}$$

$$\left(\begin{array}{cccc|cccc}1&2&1&-3&0&1&0&0\\0&-1&-2&9&0&-2&0&1\\0&0&2&-22&0&4&1&-4\\0&0&0&0&1&-1&\frac{1}{2}&-2\end{array}\right)$$

A^{-1} DNE

$$E_5=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&-\frac{1}{2}&1\end{pmatrix}$$

2. Consider the system of linear equations $\begin{aligned} 2x_2 - x_3 + x_4 &= -3 \\ 2x_1 + x_2 - 3x_3 + x_4 &= 1 \\ x_1 - 2x_2 + x_3 - 5x_4 &= 0 \end{aligned}$. Solve this system of linear equations and the corresponding homogeneous system of linear equations. (6 points)

$$\left(\begin{array}{cccc|c} 0 & 2 & -1 & 1 & -3 \\ 2 & 1 & -3 & 1 & 1 \\ 1 & -2 & 1 & -4 & 0 \end{array}\right) \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 1 & -4 & 0 \\ 2 & 1 & -3 & 1 & 1 \\ 0 & 2 & -1 & 1 & -3 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 1 & -4 & 0 \\ 0 & 5 & -5 & 9 & 1 \\ 0 & 2 & -1 & 1 & -3 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 1 & -4 & 0 \\ 0 & 1 & -1 & \frac{9}{5} & \frac{1}{5} \\ 0 & 2 & -1 & 1 & -3 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 1 & -4 & 0 \\ 0 & 1 & -1 & \frac{9}{5} & \frac{1}{5} \\ 0 & 0 & 1 & \frac{-13}{5} & \frac{-17}{5} \end{array}\right) \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

HP: $\begin{pmatrix} 15 \\ 4 \\ 13 \\ 5 \end{pmatrix}$ NP $\begin{pmatrix} 0 \\ -\frac{12}{5} \\ -\frac{4}{5} \\ 1 \end{pmatrix}$

Soln: $c\begin{pmatrix} 15 \\ 4 \\ 13 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ -\frac{12}{5} \\ -\frac{4}{5} \\ 1 \end{pmatrix}$

3. Let $\bar{u}_1 = \begin{pmatrix} 1 \\ -1 \\ 2 \\ -2 \end{pmatrix}, \bar{u}_2 = \begin{pmatrix} 0 \\ -2 \\ 1 \\ 1 \end{pmatrix}, \bar{u}_3 = \begin{pmatrix} -2 \\ 2 \\ 1 \\ -3 \end{pmatrix}$. Show whether or not there are real numbers c_1, c_2, c_3 so that

$c_1\bar{u}_1 + c_2\bar{u}_2 + c_3\bar{u}_3 = \begin{pmatrix} 3 \\ -3 \\ 1 \\ 1 \end{pmatrix}$. (6 points)

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ -1 & -2 & 2 & -3 \\ 2 & 1 & 1 & 1 \\ -2 & 1 & -3 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & -2 & 0 & 0 \\ 0 & 1 & 5 & -5 \\ 0 & 1 & -7 & 7 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 5 & -5 \\ 0 & 1 & -7 & 7 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 5 & -5 \\ 0 & 0 & -7 & 7 \end{array}\right) \quad \begin{array}{l} c_1 = 1 \\ c_2 = 0 \\ \left.\begin{array}{l} \\ \\ \end{array}\right\} c_3 = -1 \end{array}$$

4. Let D be a diagonal matrix and A a matrix. Assuming A is a correct type matrix, give formulas in terms of the rows (A_{i*}) and columns (A_{*j}) of A for AD and DA. Does AD=DA? What kind of matrix is A? (You MUST show your work for this problem. Write out some steps.) (3 points)

$$\begin{pmatrix} A_{1*} \\ A_{2*} \\ \vdots \\ A_{m*} \end{pmatrix} \begin{pmatrix} d_1 & 0 & 0 & \cdots & 0 \\ 0 & d_2 & 0 & \cdots & 0 \\ 0 & 0 & d_3 & 0 \cdots & 0 \\ \vdots & & & & \\ 0 & \cdots & & 0 & d_n \end{pmatrix} = \begin{pmatrix} d_1 A_{*1} & d_2 A_{*2} & \cdots & d_n A_{*n} \end{pmatrix}$$

$A \in M_{m \times n}$

$$\begin{pmatrix} d_1 & 0 & 0 & \cdots & 0 \\ 0 & d_2 & 0 & \cdots & 0 \\ 0 & 0 & d_3 & 0 \cdots & 0 \\ \vdots & & & & \\ 0 & \cdots & & 0 & d_n \end{pmatrix} \begin{pmatrix} A_{*1} & A_{*2} & \cdots & A_{*m} \end{pmatrix}$$

$$= \begin{pmatrix} d_1 A_{1*} \\ d_2 A_{2*} \\ \vdots \\ d_n A_{n*} \end{pmatrix}$$

$A \in M_{n \times m}$

5. Let A be a matrix. Give alternate formulas for $(A^n)^{-1}$ and $(A^n)^T$ for $n \in Z$. For example: $(A^2)^{-1} = (A^{-1})^2$ and what is $(A^{-2})^{-1}$? Explain what kind of matrix A has to be for this to make sense. (You MUST show your work for this problem. Write out some steps.) (3 points)

$$(A^3)^{-1} = (A^{-1})^3 \Rightarrow A^{-1}A^{-1}A^{-1}AAA = I$$

$$(A^{-1})^2 = A^{-1}A^{-1} = A^{-2} \Rightarrow A^{-1}A^{-1}AA = I$$

$$(A^{-1})^3 = A^{-1}A^{-1}A^{-1} = A^{-3} \Rightarrow A^{-1}A^{-1}A^{-1}AAA = I$$

$$(A^2)^T = (AA)^T = A^T A^T = (A^T)^2$$

$$(A^{-2})^T = (A^{-1}A^{-1})^T = (A^{-1})^T (A^{-1})^T = (A^T)^{-1}(A^T)^{-1} = (A^T)^{-2}$$

$A \in M_n$

6. (Bonus Point Problem – 5 points). Give the determinant of $A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ -1 & 2 & -3 & -2 & -5 \\ 2 & 4 & 6 & 8 & 9 \\ -2 & 0 & -2 & 0 & -2 \\ 3 & 6 & 0 & 10 & 14 \end{pmatrix}$.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 4 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 4 & 4 & 8 & 8 \\ 0 & 0 & -9 & -2 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 4 & 0 & 2 & 0 \\ 0 & 0 & -9 & -2 & -1 \\ 0 & 4 & 4 & 8 & 8 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \quad 1 \text{ RS}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 4 & 0 & 2 & 0 \\ 0 & 0 & -9 & -2 & -1 \\ 0 & 0 & 4 & 6 & 8 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 4 & 0 & 2 & 0 \\ 0 & 0 & -9 & -2 & -1 \\ 0 & 0 & 0 & \frac{46}{9} & \frac{28}{9} \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \Rightarrow \det = (1)(4)(-9)\left(\frac{46}{9}\right)(-1)$$

$$= 4(46) = 184$$

$\det A = -184$