MATH 238 Sochacki Exam 2
May. 30, 2014 Take Home Part 50 Points

Name KEY WEST
(Print) (1 point)

All necessary work must be shown for credit and must represent the question asked. Your work MUST be NEAT. You may NOT use computers. You may use your notes or text. All work must be your OWN work done by yourself.

I have neither received nor given help on this exam. Mon Key
(Signature) (1 points)

1. Let $A=\begin{pmatrix}1&2&3&4\\5&6&7&8\\9&10&11&12\\13&14&15&16\end{pmatrix}$. (6 points each)

(a) Give $\det(A)$

$$\begin{pmatrix}1&2&3&4\\5&6&7&8\\9&10&11&12\\13&14&15&16\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4\\0&-4&-8&-12\\0&-8&-16&-24\\0&-12&-24&-36\end{pmatrix} \to \begin{pmatrix}1&2&3&4\\0&-4&-8&-12\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

$\det(A)=0$.

(b) Give $NS(A)$ and the solution to $A\bar{x}=\begin{pmatrix}-1\\3\\7\\11\end{pmatrix}$.

$$\bar{v}_1=\begin{pmatrix}2\\-3\\0\\1\end{pmatrix} \qquad \bar{v}_2=\begin{pmatrix}1\\-2\\1\\0\end{pmatrix}$$

$NS(A)=\text{span}\{\bar{v}_1,\bar{v}_2\}$

$$\left(\begin{array}{cccc|c}1&2&3&4&-1\\5&6&7&8&3\\9&10&11&12&7\\13&14&15&16&11\end{array}\right)$$

$$\left(\begin{array}{cccc|c}1&2&3&4&-1\\0&-4&-8&-12&8\\0&-8&-16&-24&16\\0&-12&-24&-36&24\end{array}\right)$$

$$\left(\begin{array}{cccc|c}1&2&3&4&-1\\0&-4&-8&-12&8\\0&0&0&0&0\\0&0&0&0&0\end{array}\right) \qquad \bar{x}=\begin{pmatrix}3\\-2\\0\\0\end{pmatrix}$$

$$\text{soln}=c_1\begin{pmatrix}2\\-3\\0\\1\end{pmatrix}+c_2\begin{pmatrix}1\\-2\\1\\0\end{pmatrix}+\begin{pmatrix}3\\-2\\0\\0\end{pmatrix}$$

2. Show whether or not the complex numbers: $\mathbb{C} = \{a+bi \mid a,b \in \mathbb{R}, i^2 = -1\}$ under the rules

(a) $a+bi = c+di$ if and only if $a = c$ and $b = d$

(b) $(a+bi) \oplus (c+di) = (a+c)+(b+d)i$ form a vector space or not. (8 points)

(c) $c \odot (a+bi) = ca+cbi$

1) $\bar{0} = 0 + 0i$

2) $-\bar{v} = -(a+bi) = -a-bi$

3) $(a+bi) \oplus (c+di) = (a+c)+(b+d)i = (c+a)+(d+b)i = (c+di) \oplus (a+bi)$

4) $((a+bi) \oplus (c+di)) \oplus (e+fi) = (a+c)+(b+d)i \oplus (e+fi) = (a+c+e)+(b+d+f)i$
$= (a+bi) \oplus (c+e)+(d+f)i =$
$(a+bi) \oplus ((c+di) \oplus (e+fi))$

5) $c \odot ((a+bi) \oplus (e+fi)) = c \odot ((a+e)+(b+f)i) = (ca+ce)+(cb+cf)i$
$= (ca+cbi) \oplus (ce+cfi)$
$= c \odot (a+bi) \oplus c \odot (e+fi)$

6) $1 \odot (a+bi) = a+bi$

7) $(cd) \odot (a+bi) = cda + cdbi = c(da+dbi) = c \odot (d \odot (a+bi))$

8) $(c+d) \odot (a+bi) = (c+d)a + (c+d)bi = ca+da+(cb+db)i$
$= ca+cbi \oplus da+dbi = c \odot (a+bi) + d \odot (a+bi)$

3. Give the real and imaginary parts of $(1+4i)^2$ and $\frac{1}{1+i}$. (4 points)

$(1+4i)^2 = (1+4i)(1+4i) = 1+(4i)^2+4i+4i = 1+4^2i^2+8i$
$= 1-16+8i$
$= -15+8i$

$\frac{1}{1+i} \cdot \frac{1-i}{1-i} = \frac{1-i}{1^2+1^2} = \frac{1}{2} - \frac{1}{2}i$

4. Show whether the set $S=\{3-2x^2, 1-3x, 2+3x+3x^2\}$ is linearly independent or not. (6 points)

$$c_1(3-2x^2)+c_2(1-3x)+c_3(2+3x+3x^2) = 0+0x+0x^2$$

$$3c_1+c_2+2c_3+x(-3c_2+3c_3)+x^2(-2c_1+3c_3) = 0+0x+0x^2$$

$$3c_1+c_2+2c_3=0$$
$$-3c_2+3c_3=0$$
$$-2c_1+3c_3=0$$

$$\begin{pmatrix} 3 & 1 & 2 \\ 0 & -3 & 3 \\ -2 & 0 & 3 \end{pmatrix} \qquad \begin{pmatrix} 3 & 1 & 2 \\ 0 & -1 & 1 \\ -2 & 0 & 3 \end{pmatrix} \longrightarrow \begin{pmatrix} 3 & 1 & 2 \\ 0 & -1 & 1 \\ 0 & \frac{2}{3} & \frac{13}{3} \end{pmatrix}$$

$$\begin{pmatrix} 3 & 1 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 5 \end{pmatrix} \Rightarrow c_1=0, c_2=0, c_3=0$$

lin. ind.

5. Let $a,b \in \mathbb{R}$, show whether the vectors of the form $\begin{pmatrix} a \\ b-a \\ b \end{pmatrix}$ in $\mathbb{R}^3$ form a subspace or not. (6 points)

$$\begin{pmatrix} a \\ b-a \\ b \end{pmatrix} = \begin{pmatrix} a \\ -a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ b \\ b \end{pmatrix}$$
$$= a\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$a=0, b=0$ $\quad \bar{0}$ is in subspace

every vector is a lin. comb of $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

$$a_1\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + b_1\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + a_2\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + b_2\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = (a_1+a_2)\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + (a_2+b_2)\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$c\left(a\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\right) = ac\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + bc\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

6. Determine if $\begin{pmatrix}-3\\-7\\6\end{pmatrix}$ is in the span of $S=\left\{\begin{pmatrix}1\\-2\\3\end{pmatrix},\begin{pmatrix}-1\\0\\-2\end{pmatrix},\begin{pmatrix}0\\1\\1\end{pmatrix},\begin{pmatrix}4\\1\\-1\end{pmatrix}\right\}$. If it is give the coordinates in terms of S. (6 points)

$$\begin{pmatrix}-3\\-7\\6\end{pmatrix}=c_1\begin{pmatrix}1\\-2\\3\end{pmatrix}+c_2\begin{pmatrix}-1\\0\\-2\end{pmatrix}+c_3\begin{pmatrix}0\\1\\1\end{pmatrix}+c_4\begin{pmatrix}4\\1\\-1\end{pmatrix}$$

$$\downarrow\left(\begin{array}{cccc|c}1&-1&0&4&-3\\-2&0&1&1&-7\\3&-2&1&-1&6\end{array}\right)$$

$$\left(\begin{array}{cccc|c}1&-1&0&4&-3\\0&-2&1&9&-13\\0&1&1&-13&15\end{array}\right)$$

$$\left(\begin{array}{cccc|c}1&-1&0&4&-3\\0&1&1&-13&15\\0&-2&1&9&-13\end{array}\right)$$

$$\longrightarrow\left(\begin{array}{cccc|c}1&-1&0&4&-3\\0&1&1&-13&15\\0&0&3&-17&17\end{array}\right)$$

vectors are lin. dep.
inf. many solns.

$c_3=0,\ c_4=-1,\ c_2=2,\ c_1=-5$

$c_4=0,\ c_3=\frac{17}{3},\ c_2=\frac{28}{3},\ c_1=\frac{19}{3}$

7. Show whether or not the set of matrices $S=\left\{\begin{pmatrix}-2&0\\0&2\end{pmatrix},\begin{pmatrix}2&2\\0&0\end{pmatrix},\begin{pmatrix}0&-1\\2&0\end{pmatrix},\begin{pmatrix}0&0\\-2&0\end{pmatrix}\right\}$ form a basis for M_2.

(6 points)

$$c_1\begin{pmatrix}-2&0\\0&2\end{pmatrix}+c_2\begin{pmatrix}2&2\\0&0\end{pmatrix}+c_3\begin{pmatrix}0&-1\\2&0\end{pmatrix}+c_4\begin{pmatrix}0&0\\-2&0\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}$$

$-2c_1+2c_2=0 \Rightarrow c_1=c_2$

$2c_2-c_3=0 \Rightarrow c_3=2c_2=2c_1$

$2c_3-2c_4=0 \Rightarrow c_4=c_3=2c_1$

$2c_1+2c_3=0 \Rightarrow c_1+c_3=0 \Rightarrow c_3=-c_1$

$2c_1=-c_1$
$c_1=0$
$c_2=0$
$c_3=0$
$c_4=0$

one and only one soln.

Since $\dim M_2=4$ these, form a basis for M_2

Bonus Point Problem (5 points):
Consider the first order ODE $y' = 4x \sin y$. Give the equilibrium solutions of this ODE. Give a direction field for the points $(-1,-\frac{\pi}{2}),(-1,0),(-1,\frac{\pi}{2}),(0,0),(1,-\frac{\pi}{2}),(1,0),(1,\frac{\pi}{2})$. Using the first and second derivative, the equilibrium solutions and your direction field sketch a phase portrait with at least 8 possible solutions.

$y' = 4x \sin y = 0 \Rightarrow x = 0 \quad \text{OR} \quad y = k\pi \quad k \in \mathbb{Z}$

$y'' = 4\sin y + 4x\cos y \, y' = 4\sin y + 4x\cos y \, 4x\sin y$

$= 4\sin y\,(1 + 4x^2\cos y)$

$y'(-1, \frac{-\pi}{2}) = \quad \sin(\frac{-\pi}{2}) = 4$

$y'(-1; 0) = 4(-1)\sin 0 = 0$

$y'(-1; \frac{\pi}{2}) = 4(-1)\sin\frac{\pi}{2} = -4$

$y'(0; 0) = 4(0)\sin(0) = 0$

$y'(1, \frac{-\pi}{2}) = 4(1)\sin(\frac{-\pi}{2}) = -4$

$y'(1, 0) = 4(1)\sin(0) = 0$

$y'(1, \frac{\pi}{2}) = 4(1)\sin\frac{\pi}{2} = 4$