

Numerical Differentiation Homework

READ Chapter 8 in Part II of Algorithms.pdf

p.141 Exercise 1

1. Approximate $f'(x_0)$ using forward, backward and centered differences at x_0 for the given h . Approximate $f''(x_0)$ using centered differences at x_0 for the given h . Give the theoretical error estimates for each. Verify your numerical results against the theoretical results.

(a) $f(x) = \sin^2 x, x_0 = 1, h = \frac{1}{4}$ (b) $f(x) = e^{-\sin x}, x_0 = \frac{\pi}{2}, h = \frac{1}{8}$

(c) $f(x) = x \ln(x^2 + 1), x_0 = 0, h = \frac{1}{8}$

2. Fill in the following table

| x | y | FD | BD | CD | 2 nd CD |
|------|----|---------------------------|---------------------------|----|------------------------------|
| -1 | 2 | $= \frac{1-2}{-0.5-(-1)}$ | | | |
| -0.5 | 1 | | $= \frac{1-2}{-0.5-(-1)}$ | | $= \frac{1-2(1)+2}{(0.5)^2}$ |
| 0 | 1 | | | | |
| 0.5 | -1 | | | | |
| 1 | -2 | | | | |

Compare your results in the table with $f'(x_0)$ and $f''(x_0)$ for the Newton divided difference interpolating polynomial for these points.