MATH 248 – Exam 1 – October 3, 2017

Converting numbers between bases, counting, calculating, bit numbers, error, relative error, rounding chopping, root finding (zeros, solving equations), fixed point, bisection method, Fixed Point Method, Newton's Method, MATH248Part2.pdf: pp. 1-62 Analysis of Matlab Script.

1. Convert $(123.456)_7$ to base 10. (b) Convert 2/7 to base 2.
2. I have a computer that represents numbers in binary form as
   [+/-]0.d_1 d_2 d_3 d_4 d_5 d_6 d_7 x 2^k

   where d_1 = 1 and d_i = 0 or 1 for 1<i<8. What is the error and relative error in approximating 1/3 on this computer?

3. A computer stores integers on 32 bits, using the first bit for the sign and the last four bits for the exponent. What is the largest integer that can be stored on this computer?
4. A 32 bit floating number is stored with the first bit for the sign and nine bits for the exponent. What is the smallest and largest positive numbers that can be stored on this 32 bit system? What is the error and relative error in storing 1/5 on this 32 bit system?
5. If I approximate every real number by (a) chopping after the 10^{th} digit and (b) rounding at the 10^{th} digit what is the error and relative error in each case?
6. What is $(101011.0010011001100110011....)_2$ in base 10? What is the error and relative error when approximating this number with 16 digits of base 2 rounding.
7. Let x and y be real numbers. Give the error and relative error in approximating their product on a computer.
8. Set up the bisection method to approximate $\sqrt[5]{2}$. How many iterations have to be done with this algorithm to get an answer that is within 10^{-7} of $\sqrt[5]{2}$.
9. Develop an algorithm that will determine the fixed point(s) of f(x)=2x+x^3. What are the first five iterates of 2 under this algorithm? What is the error and relative error after five iterates if the initial guess is ½?
10. Show f(x) = (x+6)/(x+1) has a fixed point. For what initial guesses will the fixed point method converge to the fixed point. What is the error and relative error of your fixed point method? What is the fixed point?
11. Show f(x) = x^3+4 has a fixed point. Use the bisection method to approximate it. What is the error and relative error after 10 iterations of your algorithm? How many times do I have to use your algorithm to have an approximation with 8 digits of accuracy?
12. Show f(x) = e^(-x)-2 has a fixed point. Determine it. Give the Newton method for approximating it. Give an interval for which Newton's method is guaranteed to work.
13. Use the bisection method to approximate the fixed point of cos(x). How many times must the method be run to get an error <10^(-6)?
14. Set up Newton's Method to determine the (a) the cube root of 5 (b) 1/11 (assume your computer cannot divide).
15. Suppose $x_{k+1} = x_k^2 - 3$ is a fixed point algorithm. What are the fixed points? Set up Newton's Method to determine the fixed points.
16. Give the fixed points of $f(x) = \dfrac{1}{x-1}$. Use FPM and NM to generate four iterates using $x_1 = 1.5$