Some Exam 1 Topics

1. $S = 1 + r + r^2 + ... + r^k = \frac{1 - r^{k+1}}{1 - r}$ (If $r = 2$ then $S = 2^{k+1} - 1$)
2. $T = r + r^2 + ... + r^k = r\frac{1 - r^k}{1 - r}$ (If $r = \frac{1}{2} = 2^{-1}$ then $T = 1 - \left(\frac{1}{2}\right)^k$)
3. BM – If g is continuous on $[a,b]$ with $g(a)g(b) < 0$ then g has a root in (a,b). Dividing the interval in 2 and then using the intermediate value theorem (IVT) again gives a better approximation to the root. If n is the number of times the interval is divided in 2 and c is the root then $|x_n - c| \leq \frac{b-a}{2^n}$ where x_n is chosen by the IVT.
4. FPM - If f is continuous on $[a,b]$ and differentiable on (a,b) and f has a fixed point p in (a,b) with $|f'(p)| \leq 1$ then there is an $x_0 \in (a,b)$ near p so that $x_{k+1} = f(x_k), k = 0,1,2,...$ converges to p with $|x_{k+1} - p| \leq M^{k+1}|x_0 - p|$ and $M < 1$.
5. NM - If g is continuous on $[a,b]$ with $g(a)g(b) < 0$ then g has a root c in (a,b). If g is differentiable on (a,b) with $g'(c) \neq 0$ then there is an interval containing c such that the FPM $x_{k+1} = f(x_k) = x_k - \frac{g(x_k)}{g'(x_k)}$ converges to c for some x_0 in that interval.