

## MATH 248 – Exam 2 – October 26, 2017

### Polynomials – Maclaurin, Taylor, Newton, Lagrange, Vandermonde (Matrices, Vectors) MATH248Part2.pdf: pp. 97-125

- Newton's Method is a fixed point for determining the roots of a function  $g$ . Give the Taylor polynomial of degree 2 for the Newton method function at the root  $p$  of  $g$ . As an example do it for determining the fifth root of a real number  $a$ .
- Give a Maclaurin or Taylor polynomial of the given degree for the given function. Also give the numerical value of the last expression. (a)  $f(x) = \sin 2x, n = 8, f^{(5)}(0)$ , (b)  $f(x) = e^{-x}, n = 6, f^{(4)}(0)$   
(c)  $g(x) = \cos x^3, n = 9, f^{(6)}(0)$  (d)  $f(x) = e^{\frac{1}{x}}, n = 4, f^{(2)}(1)$ , (e)  $y = \frac{4}{1-x}, n = 8, y^{(3)}(0)$   
(f)  $g(x) = \frac{4}{1-x^2}, n = 6, g^{(4)}(0)$ , (g)  $f(x) = \sqrt{x}, n = 6, f^{(4)}(1)$
- Give the approximation given by your polynomial in question (2) at the given  $x$ . (a)  $x = 0.4$ , (b)  $x = -1$ , (c)  $x = 0.2$   
(d)  $x = 1$ , (e)  $x = \frac{1}{2}$ , (f)  $x = \frac{1}{2}$  (g)  $x = 2$
- Give an approximation to the following using an appropriate polynomial that gives 3 correct digits. (a)  $\sin 8$  (b)  $e^{-1}$  (c)  $\ln 8$   
(d)  $\frac{1}{\sqrt{5}}$
- Give the Newton polynomial, the Lagrange polynomial and the Vandermonde polynomial matrix and vector for each of the following set of points (a)  $\{(-1,0),(0,-1),(1/2,-3/4)\}$ , (b)  $\{(-1/2,9/2),(0,4),(1,6),(2,12)\}$ , (c)  $\{(-1,-10),(0,-4),(1,-2),(2,2)\}$