MATH 330 – Discrete Mathematics – Spring 2025

Turn In Homework Assignment 1

100 Points

Due: Thursday February 6, 2025

Your turn in write up will be graded on neatness, clarity of exposition (notation and definitions) and cleverness but MOSTLY correctness. There are 5 problems. Each problem is worth 20 points. You may ask me questions if you do not understand the problem. You may discuss the problems with others in class but the write up you turn in must be your OWN work. You may use the spread sheets we built in class or your own spread sheets, class notes or Chapters 1 – 3 from our textbook but your conclusions from these MUST be in your write up in your OWN words.

1. Prove

(a) The square of an even natural number is even.

(b) The square of an odd natural number is odd.

(c) Determine a pattern for $\sum_{i=0}^{n}(2i+1)$ for n a natural number. You should give the value of the sum for some values of n.

(d) From (c) determine a pattern for $\sum_{i=0}^{n} 2i$ for n a natural number. You should give the value of the sum for some values of n.

2.

(a) The word "weird" is a strange word. How many ways can you rearrange the letters of this word? How many ways can you choose a three letter string of letters from this word with no repeated letters?

(b) A certain subset of Virginia license plates has the form of three letters of the alphabet followed by four digits from the set $S = \{0,1,2,3,4,5,6,7,8,9\}$. How many license plates are in this subset? Show that $S = \{x \in W | x < 10\}$.

3.

(a) Give $(101010101010.\overline{011})_2$ in base 10.

(b) Give $\frac{3}{13}$ in base 10 in base 2.

(c) Use the geometric series formula for $\sum_{k=0}^{n} r^k$ with $r = \frac{y}{x}$ to get a formula for $(y-x)^n$. Show that your result is correct for $y = 12, x = 1$ and $n = 1,2,3,4$.

(d) Explain why the following is correct or not

$$1+\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+\cdots$$
$$=\frac{1}{1-\frac{1}{x}}$$
$$=\frac{x}{x-1}$$
$$=1-\frac{1}{1-x}$$
$$=1-(1+x+x^2+x^3+\cdots)$$
$$=-x-x^2-x^3-\cdots$$

Indicate where there is an error and explain it if there is one.

4. Show that there are as many negative odd integers as there are integers.

5. You are to determine whether or not all the people in the world can fit in the state of Texas. Use a fairly accurate approximation of the population of the world and the size of Texas. Think about how much space a person may need and the infrastructure of Texas. There is not a correct or incorrect answer but there is a good argument for there being enough land or not enough land. You might consider the current population of Texas in your write up.