HW 1 - ANSWER KEY

1(a) Let $i \in N$ then $2i$ is an even number

Let $x = 2i$ then

$$x^2 = (2i)^2 = 2^2 i^2 = 2(2i^2)$$

is divisible by 2 so x^2 is even.

(b) Let $i \in N$ then $(2i+1)$ is an odd number

Let $x = 2i+1$ then

$$x^2 = (2i+1)^2 = 2^2 i^2 + 2(2i) + 1$$
$$= 2(2i^2 + 2i) + 1$$

which is an odd natural number.

(c) $\sum_{i=0}^{n} (2i+1) = 1 + 3 + 5 + 7 + 9 + \cdots + 2n+1$

$4 = 2^2$ $n=1$

$9 = 3^2$ $n=2$

$16 = 4^2$ $n=3$

$25 = 5^2$ $n=4$

$$\sum_{i=0}^{n} (2i+1) = (n+1)^2$$

(d) $\sum_{i=0}^{n} 2i$

$$\sum_{i=0}^{n} (2i+1) = \sum_{i=0}^{n} 2i + \sum_{i=0}^{n} 1$$

$$\sum_{i=0}^{n} 2i = \sum_{i=0}^{n} (2i+1) - \sum_{i=0}^{n} 1$$
$$= (n+1)^2 - (n+1)$$
$$= n(n+1)$$

2. (a) weird

$\underline{\ 5\ }\ \underline{\ 4\ }\ \underline{\ 3\ }\ \underline{\ 2\ }\ \underline{\ 1\ }$ $\quad 5!$

$\underline{\ 5\ }\ \underline{\ 4\ }\ \underline{\ 3\ }$ $\quad 5 \cdot 4 \cdot 3 = \frac{5!}{2!}$

$= 60$

(b) $\underline{26}\ \underline{26}\ \underline{26} \mid \underline{10}\ \underline{10}\ \underline{10}\ \underline{10}$ $\quad 26^3 \cdot 10^4$

Let $S_1 = \{0, 1, 2, 3, 4, 5, 6, 7, 9)$

$S_2 = \{ x \in W \mid x < 10 \}$

Let $y \in S_1$ then $y \in W$ and $0 \le y \le 9$

So $y < 10$ and $y \in S_2$

Let $x \in S_2$ then $0 \le x \le 9$ and $x \in S_1$

3 (a) $x = (1010101010\,10.\overline{011})_2$

$= (101010101010 + .\overline{011})_2$

$= 2^{11}\ 2^9 + 2^7 + 2^5 + 2^3 + 2^1 + \frac{1}{2^2} + \frac{1}{2^3} + \frac{0}{2^4} + \frac{1}{2^5} + \frac{1}{2^6} + \cdots$

$= 2(2^{10} + 2^8 + 2^6 + 2^4 + 2^2 + 1) + \frac{3}{2^3} + \frac{3}{2^6} + \frac{3}{2^9} + \cdots$

$= 2(4^5 + 4^4 + 4^3 + 4^2 + 4 + 1) + \frac{3}{2^3}\left(1 + \frac{1}{2^3} + \left(\frac{1}{2^3}\right)^2 + \cdots\right.$

$= 2\ \frac{(1 - 4^6)}{1 - 4} + \frac{3}{2^3}\ \frac{1}{1 - \frac{1}{2^3}} = \frac{2}{3}(4^6 - 1) + \frac{3}{7}$

3(b) $\frac{3}{13} \rightarrow$ base 2

$2 \times \frac{3}{13} \quad d_{-1} = 0$

$2 \times \frac{6}{13} \quad d_{-2} = 0$

$2 \times \frac{12}{13} \quad d_{-3} = 1$

$\frac{24}{13} = 1\frac{11}{13}$

$2 \times \frac{11}{13} \quad d_{-4} = 1$

$2 \times \frac{9}{13} \quad d_{-5} = 1$

$2 \times \frac{5}{13} \quad d_{-6} = 0$

$2 \times \frac{10}{13} \quad d_{-7} = 1$

$\frac{20}{13} = 1\frac{7}{13}$

$2 \times \frac{7}{13} \quad d_{-8} = 1$

$2 \times \frac{1}{13} \quad d_{-9} = 0$

$2 \times \frac{2}{13} \quad d_{-10} = 0$

$2 \times \frac{4}{13} \quad d_{-11} = 0$

$2 \times \frac{8}{13} \quad d_{-12} = 1$

$\frac{16}{13} = 1\boxed{\frac{3}{13}}$ *

$$(0.\overline{001110110001})_2$$

(c) $\sum_{k=0}^{n} r^k = \frac{1-r^{n+1}}{1-r} = 1 + r + r^2 + \cdots + r^n$

$$(1 - r^{n+1}) = (1-r)(1 + r + r^2 + \cdots + r^n)$$

$r = \frac{y}{x} \quad \left(1 - \left(\frac{y}{x}\right)^{n+1}\right) = \left(1 - \frac{y}{x}\right)\left(1 + \frac{y}{x} + \frac{y^2}{x^2} + \cdots + \frac{y^n}{x^n}\right)$

$$\left(\frac{x^{n+1} - y^{n+1}}{x^{n+1}}\right) = \left(\frac{x-y}{x}\right)\left(1 + \frac{y}{x} + \frac{y^2}{x^2} + \cdots + \frac{y^n}{x^n}\right)$$

$$\left(\frac{x^{n+1} - y^{n+1}}{x^n}\right) = x\left(\frac{x-y}{x}\right)\left(1 + \frac{y}{x} + \frac{y^2}{x^2} + \cdots + \frac{y^n}{x^n}\right)$$

$$x^{n+1} - y^{n+1} = (x-y)\,x^n\left(1 + \frac{y}{x} + \frac{y^2}{x^2} + \cdots + \frac{y^n}{x^n}\right)$$

$$x^{n+1} - y^{n+1} = (x-y)(x^n + yx^{n-1} + \cdots + y^n)$$

OOPS!

(d) if $x = 2$ then

$$1+\frac{1}{x}+\frac{1}{x^2}+\cdots = 1+\frac{1}{2}+\frac{1}{2^2}+\cdots = \frac{1}{1-\frac{1}{2}} = 2$$

but

$$-x-x^2-x^3-\cdots = -2-2^2-2^3-\cdots \text{ D.N.E.}$$

Error

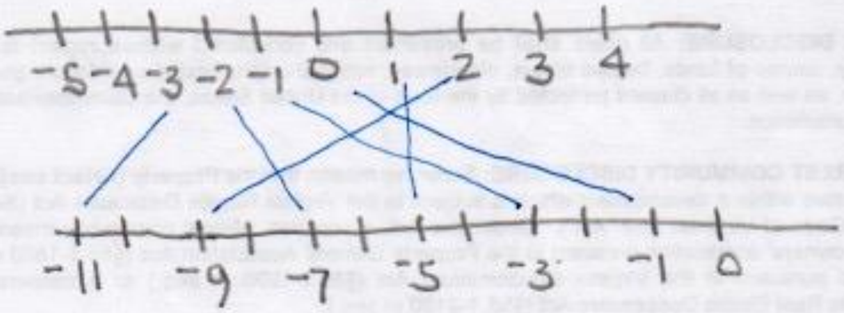
$$1+\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+\cdots = \frac{1}{1-\frac{1}{x}} \text{ only if } \left|\tfrac{1}{x}\right|<1$$

$$1<|x|$$

However,

$$1-\frac{1}{1-x} = 1-(1+x+x^2+\cdots) \text{ only if } |x|<1$$

4.



$-1 \to 0 \quad -3 \to -1 \quad -5 \to 1$

$-7 \to -2 \quad -9 \to 2$

$-11 \to -3 \quad -13 \to 3$

$-(4n-1) \to -n \quad -(4n+1) \to n \quad n \in \mathbb{N}$

$\leftarrow \quad \leftarrow \quad \leftarrow$

$0 \to -1 \quad -n \to -(4n-1) \quad n \to -(4n+1)$

5. Area of Texas in square feet is about

$$7.48804 \times 10^{12}$$

Population of world is about

$$8.25 \times 10^{9}$$

If half of Texas is available for the people of the world then

$$\frac{\text{sq. ft}}{\text{person}} \text{ is about } \frac{3.74402 \times 10^{12}}{8.25 \times 10^{9}}$$

$$\approx 450 \ \frac{\text{sq. ft}}{\text{person}}$$

$$\approx 21 \times 21 \text{ for each person}$$