MATH 330 – Discrete Mathematics – Spring 2025

Turn In Homework Assignment 2

100 Points

Due: Tuesday February 18, 2025

Your turn in write up will be graded on neatness, clarity of exposition (notation and definitions) and cleverness but MOSTLY correctness. There are 5 problems. Each problem is worth 20 points. You may ask me questions if you do not understand the problem. You may discuss the problems with others in class but the write up you turn in must be your OWN work. You may use the spread sheets we built in class or your own spread sheets, class notes or Chapters 1 - 4 from our textbook and Desmos but your conclusions from these MUST be in your write up in your OWN words. You can include tables from spread sheets and graphs from Desmos in your write up. Your write up must be turned in class and be stapled in the left corner if it is more than one page.

1. Develop a Newton's Method algorithm to determine the zeroes of sine. Give the first 10 iterates for (a) $x_0 = 1$ (b) $x_0 = 1.25$ (c) $x_0 = 1.5$ (d) $x_0 = 1.75$ (e) $x_0 = 2$.

Explain what you observe remembering to including patterns and strangeness.

2. Show that the following functions have a fixed point and give the fixed point. (You may have to iterate to give it.) Give the next ten iterates under the fixed point method for the given x_0 .

(a)
$$g(x) = |2x - 1|, x_0 = \frac{1}{4}, x_0 = \frac{4}{5}$$
 (b) $p(x) = 4x(1 - x), x_0 = \frac{1}{4}, x_0 = \frac{1}{8}$
(c) $f(x) = \sin \pi x, x_0 = \frac{1}{4}, x_0 = \frac{1}{2}$

Be sure to discuss converging to the fixed value or not.

3. Let $f(x) = A(\tan^{-1}(x) + B)$. Determine A, B so that $f: \mathbb{R} \to (-k, k)$ for any $k \in \mathbb{N}$. Give the function for at least two k. Explain what this shows. Hint: Determine the inverse of f.

4. Do the following divisions exactly as shown and give the pattern from your division and the equivalent statement for the division.

(a) $1 + x \overline{1}$ (b) $x + 1 \overline{1}$ (c) $1 - x - x^2 \overline{1}$ (d) $-x^2 - x + 1 \overline{1}$

5. Let $G_0 = a, G_1 = b$. (a) Determine the pattern in the sequence given by $G_{k+1} = G_k + G_{k-1}$. Be sure to explain it. (b) Give $\lim_{n \to \infty} \frac{G_n}{G_{n-1}}$. Does a or b affect this limit?