MATH 330 HW 3 KEY

1. Approximate $\frac{1}{\sqrt{7}}$

a) $x = \sqrt{7}$

$x^2 = 7$

Babylonian

$x^2 + x = 7 + x$

$x(x+1) = 7 + x$

$x = \frac{7+x}{1+x}$

Newton

$f(x) = x^2 - 7$

$f'(x) = 2x$

$g(x) = x - \frac{x^2-7}{2x}$

$= \frac{1}{2}x + \frac{7}{2}\frac{1}{x}$

Now

$x = \frac{1}{\sqrt{7}}$

$\frac{1}{x} = \sqrt{7}$

$f(x) = \frac{1}{x} - \sqrt{7}$ $\quad f'(x) = \frac{-1}{x^2}$

$g(x) = x - \frac{\frac{1}{x} - \sqrt{7}}{\frac{-1}{x^2}}$

$= x + x - \sqrt{7}x^2$

$= 2x - \sqrt{7}x^2$

Use approximation for $\sqrt{7}$.

b) $x = \frac{1}{\sqrt{7}}$

$\sqrt{7}x = 1$

$7x^2 = 1$

$f(x) = 7x^2 - 1$ $\quad f'(x) = 14x$

$g(x) = x - \frac{7x^2-1}{14x}$

$= \frac{1}{2}x + \frac{1}{14}\frac{1}{x}$

2. $S_{K+1} = 2S_K + 3S_{K-1}$

Suppose $S_K = cr^K$

$cr^{K+1} = 2cr^K + 3cr^{K-1}$

$cr^2 = 2cr + 3c \Rightarrow r^2 = 2r + 3$

$r^2 - 2r - 3 = 0 \Rightarrow (r-3)(r+1) = 0$

$r = 3, -1$

$S_K = c_1 3^K + c_2(-1)^K$

(i) $S_0 = c_1 + c_2 = 0 \quad c_1 = -c_2$

$S_1 = 3c_1 - c_2 = 1 \quad 3c_1 + c_1 = 1 \quad c_1 = \frac{1}{4}, c_2 = -\frac{1}{4}$

$S_K = \frac{1}{4} 3^K - \frac{1}{4}(-1)^K$

(ii) $S_0 = c_1 + c_2 = 1 \Rightarrow 4c_1 = 2 \quad c_1 = \frac{1}{2}, c_2 = \frac{1}{2}$

$S_1 = 3c_1 - c_2 = 1$

$S_K = \frac{1}{2} 3^K + \frac{1}{2}(-1)^K$

$\frac{S_{K+1}}{S_K} = 2 + 3\frac{S_{K-1}}{S_K}$

$r = 2 + 3\frac{1}{r}$

$r^2 = 2r + 3 \Rightarrow r = 3, -1$

$r = 3$

$\frac{S_{K+1}}{S_K} \to 3$

3.

1	2^0
1 1	2^1
1 2 1	2^2
1 3 3 1	2^3
1 4 6 4 1	2^4
1 5 10 10 5 1	2^5
1 6 15 20 15 6 1	2^6

$11^0 = 1$
$11^1 = 11$
$11^2 = 121$
$11^3 = 1331$
$11^4 = 14641$
$11^5 = 161,051$
$11^6 = 1,771,561$

4. $S = \{a, b, c, d, e\}$

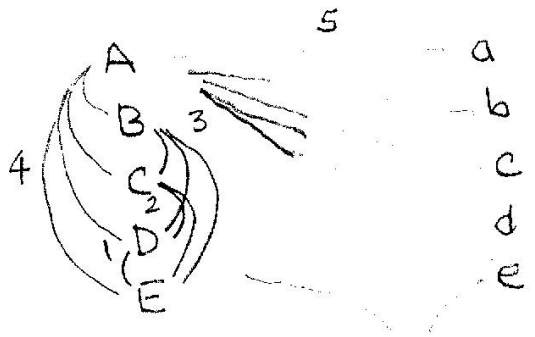
ϕ; $\{a\}, \{b\}, \{c\}, \{d\}, \{e\}$; 5

$\{a,b\}, \{c,b\}, \{d,b\}, \{e,b\}$
$\{a,c\}, \{d,c\}, \{e,c\}$
$\{a,d\}, \{e,d\}$
$\{a,e\}$; 10

$\{a,b,c\}, \{d,b,c\}, \{e,b,c\}$
$\{a,c,d\}, \{e,c,d\},$
$\{a,c,e\},$
$\{a,d,e\}$
$\{a,e,b\}$
$\{b,c,d\}$
$\{b,d,e\}$; 10

$\{a,b,c,d\}, \{e,b,c,d\}, \{a,c,d,e\},$
$\{a,b,c,e\}, \{a,b,d,e\}$; 5 $\{a,b,c,d,e\}$

5.



$2 \times (1+2+3+4) \times 2 + 5 \times 5$

$2^2 \times \left(\frac{4(5)}{2}\right) + 5^2$

$2 \times 4(5) + 5^2$

$2 \times (1+2+3+\cdots+n) + 2 \times (1+2+3+\cdots+m) + nm$

$n(n+1) + m(m+1) + nm$

$n^2 + m^2 + nm + n + m$