MATH 330 HW4 KEY

1. $r = 1 + \frac{1}{r}$

$r^2 = r + 1$

$r^2 - r - 1 = 0$

$r = \frac{1 \pm \sqrt{5}}{2}$

$r_1 = \frac{1-\sqrt{5}}{2}, \; r_2 = \frac{1+\sqrt{5}}{2}$

a) $r_1 r_2 = \frac{1-\sqrt{5}}{2} \cdot \frac{1+\sqrt{5}}{2}$

$= \frac{1-5}{4} = -1$

b) $r_1 + r_2 = 1$

c) $r_1 - r_2 = -\sqrt{5}$

(d) $\frac{r_1}{r_2} = \frac{1-\sqrt{5}}{1+\sqrt{5}} \cdot \frac{1-\sqrt{5}}{1-\sqrt{5}}$

$= \frac{6-2\sqrt{5}}{-4}$

$= \frac{\sqrt{5}-3}{2}$

(e) $\frac{r_2}{r_1} = \frac{1+\sqrt{5}}{1-\sqrt{5}} \cdot \frac{1+\sqrt{5}}{1+\sqrt{5}}$

$= \frac{6+2\sqrt{5}}{-4} = -\frac{3+\sqrt{5}}{2}$

2. $\frac{a}{b} = 1 + \frac{a}{b}$

$\frac{a}{b} = 1 + \frac{b}{a} \; (ab)$

$a^2 = ab + b^2$

$a^2 - b^2 = ab$

(f) $\frac{1+\sqrt{5}}{2} = \frac{a}{10}$

$a = 5(1+\sqrt{5})$ or $5(1-\sqrt{5})$

(g) $\frac{1+\sqrt{5}}{2} = \frac{a}{b} = \frac{10}{b}$

$b = \frac{20}{1+\sqrt{5}} \cdot \frac{1-\sqrt{5}}{1-\sqrt{5}}$

$= -5(1-\sqrt{5})$

$= 5(\sqrt{5}-1)$ or

$= -5(\sqrt{5}+1)$

(h) $a^2 + (a+b)^2 = d^2$

$a^2 + a^2 + 2ab + b^2 = d^2$

$2a^2 + 2ab + b^2 = d^2$

$2ab + b^2 + 2ab + b^2 = d^2$

$2b^2 + 4ab = d^2$

$2b(b+2a) = d^2$

(i) $a^2 + ab = 2ab + b^2$

2. $f(x) = x^3 - x - 1$

(a) $f(x) = 0$ $\quad f'(x) = 3x^2 - 1 \quad g(x) = x - \frac{x^3 - x - 1}{3x^2 - 1}$

NM: $x_{k+1} = x_k - \frac{x_k^3 - x_k - 1}{3x_k^2 - 1}$

See Graph and Table

(b) $f(x) = x \qquad x^3 - x - 1 = x \Rightarrow x^3 - 2x - 1 = 0$

$g(x) = x^3 - 2x - 1 \quad g'(x) = 3x^2 - 2$

NM: $x_{k+1} = x_k - \frac{x_k^3 - 2\cancel{x_k} - 1}{3x_k^2 - 1}$

See Graph and Table

(c) $r = 1 + \frac{1}{r} \Rightarrow r^2 = r + 1$

$r^2 - r - 1 = 0$

$x^3 - x - 1 = 0$

$r^2 - r - 1 = 0$ has 2 roots $\frac{1 \pm \sqrt{5}}{2}$

$\approx -0.618, 1.618$

$x^3 - x - 1 \doteq 0$ has 1 root. *

≈ 1.3247

has 3 fixed points $-1, \frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}$

3. (a) $\sum_{K=0}^{10} 11^K = \frac{11^{11}-1}{11-1} = 28531167061$ AVG $= \frac{\quad}{11} = 2593742460$

(b) $5+10+15+\dots+1000 = 5(1+2+3+\dots+200)$

$= 5 \ \frac{200(200+1)}{2} = 100,500$

AVG $= \frac{100,500}{200}$

$= 502,5$

(c) $\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\dots+\frac{1}{1048756}$

$= \frac{1}{4}\left(1+\frac{1}{2}+\left(\frac{1}{2}\right)^2+\dots+\left(\frac{1}{2}\right)^{18}\right)$

$= \frac{1}{4} \ \frac{1-\left(\frac{1}{2}\right)^{19}}{1-\left(\frac{1}{2}\right)} \cong \frac{0.9998}{2} \approx 0.5$ AVG $\cong \frac{0.5}{19}$

(d) $1+3+9+11+17+19+\dots+201+203 =$

$1+9+17+25+\dots+201+$

$3+11+19+27+\dots+203$

$= \frac{202(26)}{2} + \frac{206(26)}{2} = 5304$

AVG $= \frac{5304}{52} = \frac{1326}{13} = 102$

4. 5% yearly $= \frac{0.05}{12} \cong 0.0042 = r$

a) $X_0 = 1000$

$$X_1 = X_0(1+r)$$

$$X_2 = X_1(1+r) = X_0(1+r)^2$$

$$X_{K+1} = f(X_K) = (1+r)X_K = X_0(1+r)^K$$

$$X_{120} = 1000\left(1+\frac{0.05}{12}\right)^{120}$$

$$\approx 1647.$$

(b) $X_0 = 50$

$$X_1 = X_0(1+r) + X_0$$

$$X_2 = X_1(1+r) + X_0 = X_0(1+r)^2 + X_0(1+r) + X_0$$

$$= X_0\left[(1+r)^2 + (1+r) + 1\right]$$

$$X_{K+1} = f(X_K) = X_K(1+r) + X_0$$

$$= X_0\left[(1+r)^{K+1} + (1+r)^K + \dots + (1+r) + 1\right]$$

$$= X_0\,\frac{(1+r)^{K+2}-1}{(1+r)-1} = X_0\,\frac{(1+r)^{K+2}-1}{r}$$

$$X_{120} = 50\,\frac{\left(1+\frac{0.05}{12}\right)^{121}-1}{\frac{0.05}{12}} \approx 7846.46$$

5. (a) $n = 3$ $\quad x_1 = -1,\ x_2 = 0,\ x_3 = 1$

$$\text{AVG VAL} \approx \frac{f(-1) + f(0) + f(1)}{3} = \frac{e^{-1} + e^{0} + e^{1}}{3}$$

$$\approx$$

(b) $n = 5$ $\quad x_1 = -1,\ x_2 = \frac{1}{2},\ x_3 = 0,\ x_4 = \frac{1}{2},\ x_5 = 1$

$$\text{AVG VAL} \approx \frac{1}{5} \sum_{k=1}^{5} f(x_i) \approx$$

(c) $n = 7$ $\quad \Delta x = \frac{x_7 - x_1}{6} = \frac{1+1}{6} = \frac{1}{3}$

$$x_i = x_1 + (i-1)\Delta x$$

$$\text{AVG VAL} \approx \frac{1}{7} \sum_{k=1}^{7} f(x_i) \approx$$

(d) $n = 9$ $\quad \Delta x = \frac{1}{4}$ $\quad x_i = x_1 + (i-1)\Delta x$

$$\text{AVG VAL} \approx \frac{1}{9} \sum_{k=1}^{9} f(x_i) \approx$$

(e) $\frac{1}{b-a} \int_a^b f(x)\,dx$

$$\int_{-1}^{1} e^{-x^2}\,dx \approx 2$$

See Sheet