

MATH 330 – Discrete Mathematics – Spring 2025

Turn In Homework Assignment 6

100 Points

Due: Tuesday April 8, 2025

Your turn in write up will be graded on neatness, clarity of exposition (notation and definitions) and cleverness, but MOSTLY correctness and using the best counting principles. There are 5 problems. Each problem is worth 20 points. You may ask me questions if you do not understand the problem. You may discuss the problems with others in class but the write up you turn in must be your OWN work. You may use the spread sheets we built in class or your own spread sheets, class notes or Chapters 1 – 7 from our textbook and Desmos but your conclusions from these MUST be in your write up in your OWN words. You can include tables from spread sheets and graphs of functions from Desmos in your write up. Your write up must be turned in class and be stapled in the left corner if it is more than one page.

1. Write down Pascal's triangle to the row for a set with 7 elements. Next to it write down the Pascal triangle using C_k^n notation correctly. From this give ten rules for combinations using C_k^n from this Pascal triangle according to the rules given in class.
2. There are 8 people in a room with 3 doors. In how many ways can these 8 people leave the room? Be sure to explain your thought process using counting principles.
3. Given a standard deck of 52 cards, answer the following questions. (a) How many 13 card hands are possible? (b) How many 5 card hands have only numbers in them? (c) You are given 3 aces in a row. What is the probability the 5th card is an ace? (d) You have 5 cards face down. What is the probability one is an ace, one is a king, one is a queen and one is a jack? (e) What is the probability of 3 cards being the same (except for suit) if you are given 7 cards at random?
4. Given 5 dice with exactly one of 1 – 6 dots on each side (standard dice), answer the following questions. (a) What is the probability of getting a permutation of {1,2,3,4,5}? (b) What is the probability of getting a sum of 18 if 1 of the dice is a 6? (c) What is the probability of getting the product 32? (d) What is the probability of at exactly 2 of the dice being even? (e) What is the probability of at least 4 of the dice being odd?
5. Given the two permutations $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 1 & 5 & 2 \end{bmatrix}$, $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 3 & 5 & 4 \end{bmatrix}$. (a) What is $\beta(\alpha(1))$? (b) What is $\alpha(\beta(1))$? (c) What is $\alpha \circ \beta$ in matrix form? (d) What is $\beta \circ \alpha$ in function form? (e) What is the inverse of α in function form? (f) What is the inverse of β in matrix form? (g) What is the inverse of $\alpha \circ \beta$ in matrix form? (h) What is $\beta^{-1} \circ \alpha^{-1}$ in function form? (i) How many permutation operators of {1,2,3,4,5} start with 1,2? (j) What is the probability of choosing a permutation operator that does not start with 1,2?