MATH 330 HW 6 KEY

1.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

C^0_0
$C^1_0 \quad C^1_1$
$C^2_0 \quad C^2_1 \quad C^2_2$
$C^3_0 \quad C^3_1 \quad C^3_2 \quad C^3_3$
$C^4_0 \quad C^4_1 \quad C^4_2 \quad C^4_3 \quad C^4_4$
$C^5_0 \quad C^1_1 \quad C^5_2 \quad C^5_3 \quad C^5_4 \quad C^5_5$
$C^6_0 \quad C^6_1 \quad C^6_2 \quad C^6_3 \quad C^6_4 \quad C^6_5 \quad C^6_6$
$C^7_0 \quad C^7_1 \quad C^7_2 \quad C^7_3 \quad C^7_4 \quad C^7_5 \quad C^7_6 \quad C^7_7$

$$C^{2n+1}_{K} = C^{2n+1}_{2n+1-K}$$
$$C^{2n}_{K} = C^{2n}_{2n-K}$$

$$\sum_{K=0}^{n} C^n_K = 2^n$$

2.

$(8,0,0) \to C^3_1 = 3 = \frac{3!}{2!}$
$(7,1,0) \to 3! = 6$
$(6,2,0) \to 3! = 6$
$(6,1,1) \to C^3_1 = 3 = \frac{3!}{2!}$
$(5,3,0) \to 3! = 6$
$(5,2,1) \to 3! = 6$
$(4,4,0) \to C^3_1 = 3 = \frac{3!}{2!}$
$(4,3,1) \to 3! = 6$
$(4,2,2) \to C^3_1 = 3 = \frac{3!}{2!}$
$(3,3,2) \to C^3_1 = 3 = \frac{3!}{2!}$

$5 \times 3 + 5 \times 6$

45

3 (a) $C^{52}_{13} = 635,013,559,600 \approx 600$ Billion

(b) $52 - 4 \cdot 4 = 36$ $\quad C^{36}_{5} = 376,992$

A, K, Q, J

(c) $\frac{A}{52} \; \frac{A}{51} \; \frac{A}{50} \; \frac{\overset{\text{not an}}{A}}{49} \; \frac{A}{48}$ $\quad 1/48 \approx 0.02$

(d) $\frac{A}{4} \; \frac{K}{4} \; \frac{Q}{4} \; \frac{J}{4} \; \frac{\text{any}}{48}$ $\quad \dfrac{4^4 \cdot 48}{C^{52}_{5}} \approx 0.0047$

(e) $\underline{*} \; \underline{*} \; \underline{*} \; / \; \underline{\;} \; \underline{\;} \; \underline{\;} \; \underline{\;}$

$C^{13}_{1} \cdot C^{4}_{3}$ $\quad C^{49}_{4}$

52

$$\frac{52 \cdot C^{49}_{4}}{C^{52}_{7}} \approx 0.0824$$

4 (a) permutations of $(1,2,3,4,5) = 5!$

$$\frac{5!}{6^5} \doteq 0.0154$$

4 (b)

$\underline{6}\ \underbrace{\_\_\ \_\_\ \_\_\ \_\_}_{12}$

42
72
12
1
———
123

$(6,4,1,1) \qquad \frac{4!}{2!} = 12$

$(6,3,2,1) \qquad 4! = 24$

$(5,5,1,1) \qquad \frac{4!}{2!\,2!} = C^4_2 = 6$

$(5,4,3,1) \qquad 4! = 24$

$(5,4,2,2) \qquad \frac{4!}{2!} = 12$

$(5,3,3,1) \qquad \frac{4!}{2!} = 12$

$(5,3,2,2) \qquad \frac{4!}{2!} = C^4_2 = 12$

$(4,4,3,1) \qquad \frac{4!}{2!} = 12$

$(4,4,2,2) \qquad \frac{4!}{2!\,2!} = C^4_2 = 6$

$(4,3,3,2) \qquad \frac{4!}{2!} = 12$

$(3,3,3,3) \qquad \underline{1}$

$\frac{123}{6^4} \approx 0.095$

5 choices for 6

≈ 0.475

4 (c) ___ ___ ___ ___ ___

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32 \rightarrow 1$$

$$2 \cdot 2 \cdot 2 \cdot 4 \cdot 1 = 2^3 \cdot 4 \cdot 1 = 32 \rightarrow \frac{5!}{3!}$$

$$2 \cdot 4 \cdot 1 \cdot 4 \cdot 1 = 2 \cdot 4^2 \cdot 1 = 32 \rightarrow \frac{5!}{2!2!}$$

$$\frac{1 + 5 \cdot 4 + 5 \cdot 3 \cdot 2}{6^5} = \frac{51}{6^5} \approx 0.0066$$

(d) $\underline{E_1}\ \underline{E_2}\ \underline{O_1}\ \underline{O_2}\ \underline{O_3}$

5 for E_1, 4 for E_2, 3 even (2,4,6)
3 odd (1,3,5)

$$\frac{C^5_2 \cdot 3^2 \cdot 3^3}{6^5} = 0.3125$$

(e) $\underline{O_1}\ \underline{O_2}\ \underline{O_3}\ \underline{O_4}\ \underline{O_5}$ $\quad 3^5$

$\underline{O_1}\ \underline{O_2}\ \underline{O_3}\ \underline{O_4}\ \underline{E_1}$ $\quad 5 \cdot 3^4$

$$\frac{3^5 + 5 \cdot 3^5}{6^5} = 0.1875$$

5. $\alpha = (3\ 4\ 1\ 5\ 2)$ $\beta = (2\ 1\ 3\ 5\ 4)$

a) $\alpha(1) = 3$ $\beta(3) = 3$ $\beta(\alpha(1)) = 3$

(b) $\beta(1) = 2$ $\alpha(2) = 4$ $\alpha(\beta(1)) = 4$

(c) $\begin{matrix} 2 & 1 & 3 & 5 & 4 \\ 4 & 3 & 1 & 2 & 5 \end{matrix}$ $\alpha \circ \beta = (4\ 3\ 1\ 2\ 5)$

(d) $\beta(\alpha(1)) = \beta(3) = 3$ $\beta(\alpha(2)) = \beta(4) = 5$

$\beta(\alpha(3)) = \beta(1) = 2$ $\beta(\alpha(4)) = \beta(5) = 4$

$\beta(\alpha(5)) = \beta(2) = 1$

(e) $\alpha(3) = 1$ $\alpha^{-1}(1) = 3$

$\alpha(5) = 2$ $\alpha^{-1}(2) = 5$

$\alpha(1) = 3$ $\alpha^{-1}(3) = 1$

$\alpha(2) = 4$ $\alpha^{-1}(4) = 2$

$\alpha(4) = 5$ $\alpha^{-1}(5) = 4$

(f) $\begin{pmatrix} 2 & 1 & 3 & 5 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$ $\beta^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 3 & 5 & 4 \end{pmatrix}$

$\beta^{-1} = \beta$

5 (g) $\alpha \circ \beta = \begin{pmatrix} 4 & 3 & 1 & 2 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$

$$(\alpha \circ \beta)^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 2 & 1 & 5 \end{pmatrix}$$

(h) $\beta^{-1} \circ \alpha^{-1} = (\alpha \circ \beta)^{-1}$

$\beta^{-1} \circ \alpha^{-1}(1) = 3$ $\beta^{-1} \circ \alpha^{-1}(2) = 4$

$\beta^{-1} \circ \alpha^{-1}(3) = 2$ $\beta^{-1} \circ \alpha^{-1}(4) = 1$

$\beta^{-1} \circ \alpha^{-1}(5) = 5$

(i) 1, 2, —, —, — $3!$

(3, 4, 5)

(j) 1 − probability of (i)

$$1 - \frac{3!}{5!} = 0.95$$