MATH 330 – Discrete Mathematics – Spring 2025

Turn In Homework Assignment 7

100 Points

Due: Thursday April 17, 2025

Your turn in write up will be graded on neatness, clarity of exposition (notation and definitions) and cleverness, but MOSTLY correctness and using the best counting principles. There are 5 problems. Each problem is worth 20 points. You may ask me questions if you do not understand the problem. You may discuss the problems with others in class but the write up you turn in must be your OWN work. You may use the spread sheets we built in class or your own spread sheets, class notes or Chapters 1 – 8 from our textbook and Desmos but your conclusions from these MUST be in your write up in your OWN words. You can include tables from spread sheets and graphs of functions from Desmos in your write up. Your write up must be turned in class and be stapled in the left corner if it is more than one page.

1. Let $\alpha = (5 \quad 4 \quad 3 \quad 2 \quad 1)$ and $\beta = (5 \quad 1 \quad 4 \quad 2 \quad 3)$. (a) Give the graphs for α, β.

(b) Show $\alpha^{-1} = \alpha$. (c) Give $(\alpha \circ \beta)^{-1}$ and its graph. (d) Give an α on S_4 so that $\alpha^{-1} = \alpha$.

(e) How many permutation operators on S_4 have $\alpha^{-1} = \alpha$?

(f) What is the probability of drawing a permutation operator from S_4 with $\alpha^{-1} = \alpha$?

2. For the following graphs give (a) the circuits or paths; if there are none, tell clearly why,

(b) the incidence matrix.

(i) (ii)

(iii)

3. Give the graphs for the following incidence matrix. If it is not an incidence matrix, tell clearly why. Also, if it is a graph tell why clearly whether or not there is a circuit or path.

(a) $A = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$ (b) $A = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$

(c) $A = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$

4. For the incidence matrix below give the number of edges and vertices and tell clearly why there is a circuit, path or neither. If there is no cycle or path, give the location of the fewest 0 or 1 locations that you can change to give either a cycle or a path.

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

5. Let $\begin{matrix} x_{k+1} = x_k + y_k \\ y_{k+1} = x_k + y_k \end{matrix}$ (a) Give the output for $(x_0, y_0) = (1,0)$. (b) Give the output for $(x_0, y_0) = (0,1)$.

(c) Give the output for $(x_0, y_0) = (1,1)$. (d) Give the matrix for this algorithm and the graph.

(e) Give the eigenvalues of the algorithm.