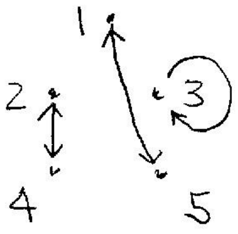
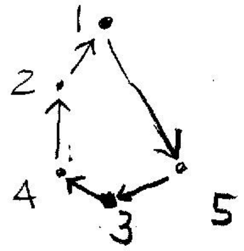
MATH 330 HW7 KEY

1. $\alpha = (5\ 4\ 3\ 2\ 1)$ $\quad \beta = (5\ 1\ 4\ 2\ 3)$

a)



b)



b) $\alpha(\alpha(1)) = \alpha(5) = 1$ $\quad \alpha(\alpha(2)) = \alpha(4) = 2$

$\alpha(\alpha(3)) = \alpha(3) = 3$ $\quad \alpha(\alpha(4)) = \alpha(2) = 4$

$\alpha(\alpha(5)) = \alpha(1) = 5$

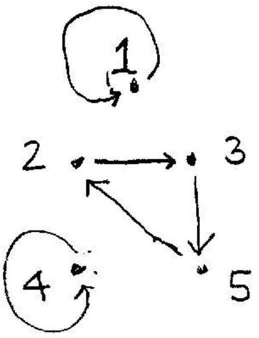
c) $(\alpha \circ \beta)^{-1} = \beta^{-1} \circ \alpha^{-1} = \beta^{-1} \circ \alpha$

$\underline{\beta^{-1}}$ $\quad \begin{pmatrix} 5 & 1 & 4 & 2 & 3 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$ $\quad (2\ 4\ 5\ 3\ 1) = \beta^{-1}$

$\underline{\beta^{-1}(\alpha)}$ $\quad \beta^{-1} = (2\ 4\ 5\ 3\ 1)$

$\alpha = (5\ 4\ 3\ 2\ 1)$

$(\alpha \circ \beta)^{-1} = (1\ 3\ 5\ 4\ 2)$



d) $\alpha^{-1} = \alpha$ $\quad \alpha = (1\ 2\ 3\ 4)$

$\alpha = (2\ 1\ 3\ 4)$

$\alpha = (1\ 2\ 4\ 3)$

$\alpha = (4\ 3\ 2\ 1)$

$\alpha = (3\ 2\ 1\ 4)$

(e)

1	__	__	__	2 1	
2	__	__	__	2	10
3	__	__	__	2	
4	__	__	__	2	

(f) $\dfrac{10}{4!} = \dfrac{5}{12}$

2 (i) More than 2 odd vertices (2, 3, 4, 5)
No circuit or path

$$\begin{array}{c|c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} & e \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} & \begin{matrix} 2 \\ 3 \\ 3 \\ 3 \\ 3 \\ 2 \end{matrix} \end{array}$$

No circuit
No path

(ii) Exactly 2 odd vertices (1, 6)
No circuit

path
(1, 2, 4, 3,
7, 6, 4, 1,
5, 6)

$$\begin{array}{c|c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix} & e \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} & \begin{matrix} 3 * \\ 2 \\ 2 \\ 4 \\ 2 \\ 3 * \\ 2 \end{matrix} \end{array}$$

(iii) Every vertex even, has a circuit

(1, 2, 6, 5, 4, 3, 5, 2, 8, 7, 3, 1)

$$\begin{array}{c} \\ 1\\2\\3\\4\\5\\6\\7\\8 \end{array}\begin{array}{c} \begin{array}{cccccccc} 1&2&3&4&5&6&7&8 \end{array} \\ \left(\begin{array}{cccccccc} 0&1&1&0&0&0&0&0\\ 1&0&0&0&1&1&0&1\\ 1&0&0&1&1&0&1&0\\ 0&0&1&0&1&0&0&0\\ 0&1&1&1&0&1&0&0\\ 0&1&0&0&1&0&0&0\\ 0&0&1&0&0&0&0&1\\ 0&1&0&0&0&0&1&0 \end{array}\right) \end{array}\begin{array}{c} e\\ \left.\begin{array}{c} 2\\4\\4\\2\\4\\2\\2\\2 \end{array}\right\} \text{Circuit} \end{array}$$

3. (a) A is symmetric (rows, columns)

e
2
2
1 *
1 *
2

No Ciruit (odd vertices)

path (exactly 2 odd vertices)

(b) A is symmetric <u>2 (ii)</u>

e
3 *
2
2
4
2
3 *
2

path

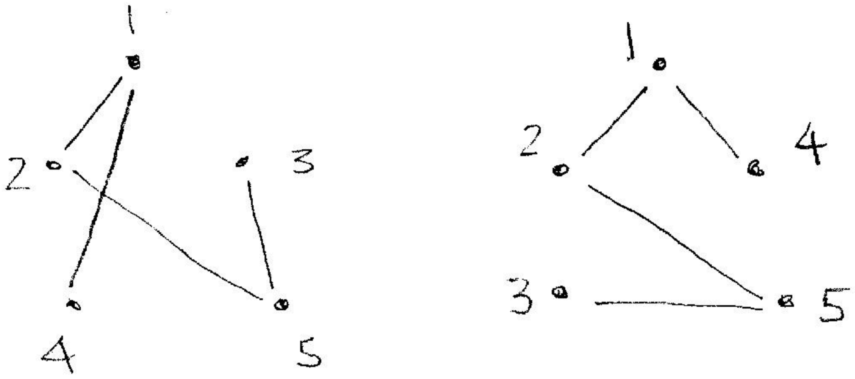
No circuit

3(c) A is symmetric

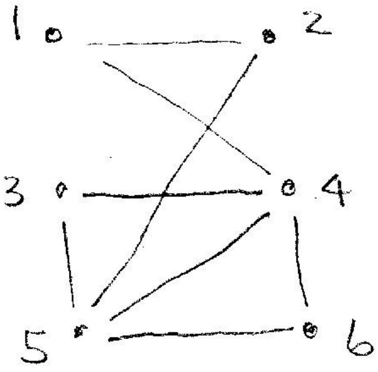
e
2
2
2
4
4
2

Circuit

3(a)



(c)



4. A is symmetric and 9x9

The number of vertices is 9.

row 1 sums to 3
row 2 sums to 4
row 3 sums to 1
row 4 sums to 5
row 5 sums to 2
row 6 sums to 3
row 7 sums to 4
row 8 sums to 2
row 9 sums to 2

sums to 26 which is even

There are 13 edges

There is NO circuit or path because there are too many odd vertices.

If we change row 3 and row 4 by 1 then we have a path.

Change $A_{3,4}$ to 1 and $A_{4,3}$ to 1 then the number of odd vertices is exactly 2. and there is a path.

If we then change row 1 and row 6, we can have a circuit. Change $A_{1,6}$ to 0 and $A_{6,1}$ to 0 then all vertices are even and there is a circuit.

5. $\begin{pmatrix} x_{k+1} \\ y_{k+1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_k \\ y_k \end{pmatrix}$

See Excel Spread Sheet

$\left.\begin{array}{l} \text{(a)} \\ \text{(b)} \\ \text{(c)} \end{array}\right\}$ Output is $\{(2^k, 2^k) \mid k \in W\}$ after (x_0, y_0).

(d) $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ 

(e) $(1-\lambda)^2 - 1 = 0$

$(1-\lambda)^2 = 1$

$1-\lambda = \pm 1$

$-\lambda = -1 \pm 1 = 0, -2$

$\lambda = 0$ or 2