

MATH 330 – Discrete Mathematics – Spring 2025

Turn In Homework Assignment 8

100 Points

Due: Tuesday April 29, 2025

Your turn in write up will be graded on neatness, clarity of exposition (notation and definitions) and cleverness, but MOSTLY correctness and using the best counting principles. There are 5 problems. Each problem is worth 20 points. You may ask me questions if you do not understand the problem. You may discuss the problems with others in class but the write up you turn in must be your OWN work. You may use the spread sheets we built in class or your own spread sheets, class notes or Chapters 1 – 8 from our textbook and Desmos but your conclusions from these MUST be in your write up in your OWN words. You can include tables from spread sheets and graphs of functions from Desmos in your write up. Your write up must be turned in class and be stapled in the left corner if it is more than one page.

1. Let $\begin{matrix} x_{k+1} = ax_k + ay_k \\ y_{k+1} = ax_k \end{matrix}$ (a) Analyze this recursion for the set $a \in \mathbb{R}$. (b) What is the output set if $a = 1$ and $(x_0, y_0) = (1, 1)$? (c) What is the output set if $a = 2$ and $(x_0, y_0) = (0, 1)$?

2. Let $A = O(x_0, f)$ where $x_0 = -1$ and $f(x) = 1 - |x|$. Let $B = \{n \in \mathbb{N} | n < 6\}$. Let $C = \{x \in \mathbb{R} | |x - 6| < 4\}$. Give the following.

(a) $|A|$ (b) $A \cap C$ (c) C^c (d) $|A \cup B|$ (e) $A \cap (B \cup C)$ (f) $(A \cap B) \cup (A \cap C)$ (g) $C - B$

(h) The cardinality of the set of all subsets of permutation operators on B .

3. Draw Venn Diagrams for each of these and if it is true give a word proof.

(a) $A^c \cap B = A - B$

(b) Give the true statement for another way you think you can express $A \cup (B \cap C)$.

4. Number the letters of the alphabet consecutively from 1 to 26. Let $A = \{\text{vowels}\}$. Let $B = \{2^k | k \in \mathbb{N} \cap [0, 4]\}$. Let $C = \{3 + 4k | k \in \{0, 1, 2, 3, 4, 5\}\}$. Let $D = \{13, 17, 25\}$. Let $E = \{2k \neq 2^k, k \in \mathbb{N} \cap (0, 13]\}$.

Explain clearly why $P = \{A, B, C, D, E\}$ is or is not a partition. That is, show why it is a partition or give all the places it fails to be a partition. Be sure to give the letters and their corresponding numerical values in your explanation.

5. There are 250 students at Happy Middle School. The students sign up for any of baseball, kickball, soccer or volleyball. The principal looks at the list and notices that 100 students total are in soccer, 25 are in soccer and volleyball, 75 students total are in baseball, 25 are in volleyball and baseball, 30 students total are in kickball, and 25 are in baseball and kickball. Give the following.

(a) A Venn Diagram with the numbers for each group of students.

(b) The number of students playing volleyball, baseball and kickball.

(c) The probability of picking a student who only plays soccer.

(d) Give a partition of the set of the 250 students in terms of the sport they are playing. Be sure to label your sets carefully. Hint: Using your Venn Diagram can help.