MATH 330 HW 8 KEY

1. (a)

$$x_{k+1} = a x_k + a y_k$$
$$y_{k+1} = a x_k$$

$$= \begin{pmatrix} a & a \\ a & 0 \end{pmatrix} \begin{pmatrix} x_k \\ y_k \end{pmatrix}$$

$$= a \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_k \\ y_k \end{pmatrix}$$

$$\frac{x_{k+1}}{y_{k+1}} = \frac{a x_k + a y_k}{a x_k} = \frac{a x_k}{a x_k} + \frac{a y_k}{a x_k}$$

$$= 1 + \frac{y_k}{x_k} \quad \text{(Fibonacci)}$$

For all $a \in R$ $\quad \frac{x_{k+1}}{y_{k+1}} \to r$

where $r = 1 + \frac{1}{r}$ OR

$\frac{y_{k+1}}{x_{k+1}} \to \frac{1}{r}$ for all $a \in R$

$$y \to \frac{1}{r} x + b$$

Let $A := \begin{pmatrix} a & a \\ a & 0 \end{pmatrix}$

Graph: 

Eigenvalues:

$$(a-\lambda)(-\lambda) - a^2 = 0$$

$$\lambda^2 - a\lambda - a^2 = 0$$

$$\lambda = \frac{a \pm \sqrt{a^2 + 4a^2}}{2}$$

$$= \frac{a \pm a\sqrt{5}}{2} = a\,\frac{1 \pm \sqrt{5}}{2}.$$

If $a = 1 \rightarrow$ golden ratio

$a = 1$, $(x_0, y_0) = (1, 1)$

(b) Output is Fibonacci Numbers

(c) $a = 2$ $\quad (x_0, y_0) = (0, 1)$

Output is $\{(2^k \cdot F_k, 2^k \cdot F_{k-1}), k \in W\}$

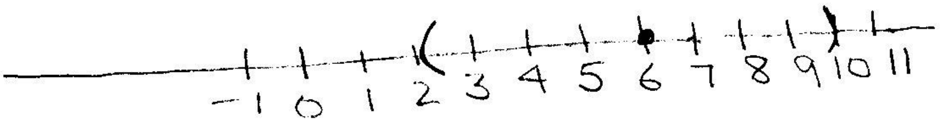
(See Excel)

2. $f(x) = 1 - |x|$

$A = O(-1, f) = \{-1, f(-1), f(f(-1)), \dots\}$
$= \{-1, 0, 1\}$

$B = \{n \in N \mid n < 6\} = \{1, 2, 3, 4, 5\}$

$C = \{x \in R \mid |x - 6| < 4\}$



$= \{x \in R \mid 2 < x < 10\}$

a) $|A| = 3$

b) $A \cap C = \phi$

c) $C^c = \{x \in R \mid x \leq 2 \text{ or } x \geq 10\}$

d) $A \cup B = \{-1, 0, 1, 2, 3, 4, 5\}$
$|A \cup B| = 7$

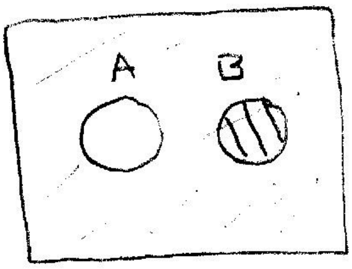
e) $A \cap (B \cup C) = \{-1, 0, 1\} \cap \{1, 2 \leq x < 10\}$
$= \{1\}$

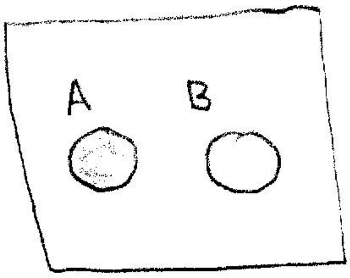
f) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C) = \{1\}$

g) $C - B = (2,3) \cup (3,4) \cup (4,5) \cup (5,10)$

h) There are $5!$ p.o. So the number of subsets is $2^{5!}$

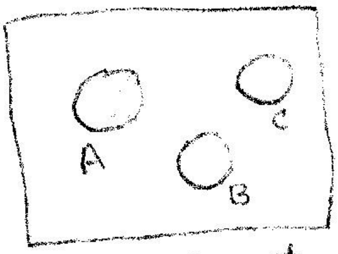
3 (a) $A^c \cap B$ $\qquad\qquad$ $A - B$





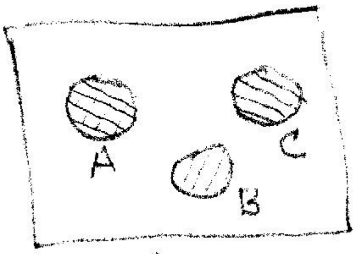
$$A^c \cap B \neq A - B$$

(b) $A \cup (B \cap C)$ $\qquad\qquad$ $(A \cup B) \cap (A \cup C)$

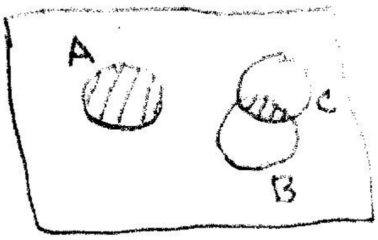


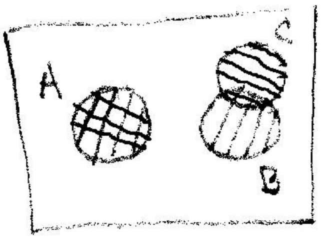
$B \cap C = \phi$

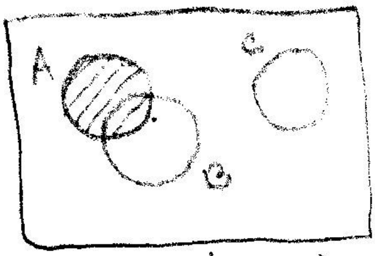
A



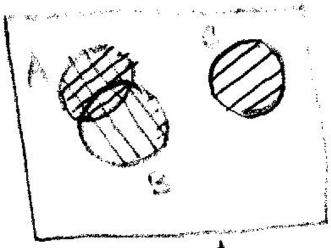
A



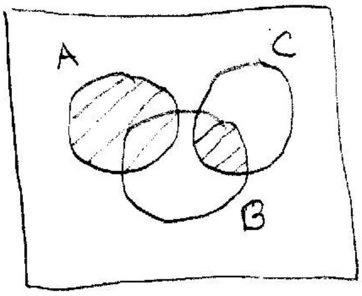


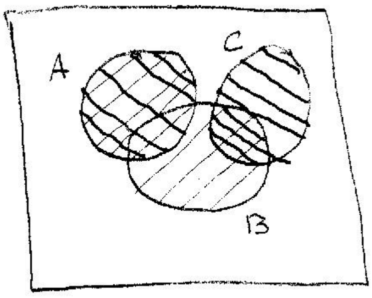


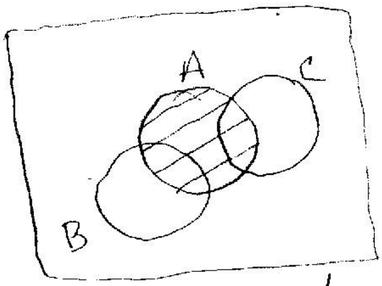
$B \cap C = \phi$ $\qquad$ A



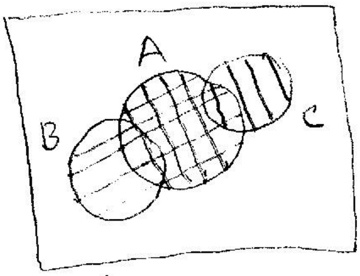
A



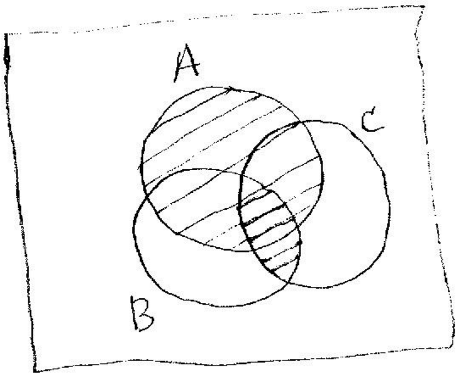


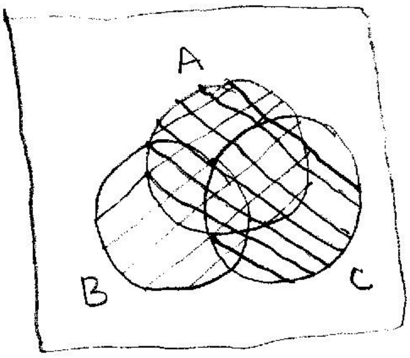


A $B \cap C = \phi$



A





$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Let $x \in A \cup (B \cap C)$

$x \in A$ or $x \in B$ and $x \in C$

$x \in A$ or $x \in B$ and $x \in A$ or $x \in C$

$x \in (A \cup B) \cap (A \cup C)$

Let $x \in (A \cup B) \cap (A \cup C)$

$x \in A$ or $x \in B$ and $x \in A$ or $x \in C$

$x \in A$ or $x \in B$ and $x \in C$

$x \in A \cup (B \cap C)$

4. See Excel

5. See Excel