Some Basic Rules, Definitions and Properties

DEF Let $n \in \mathbb{N}$ and $P = \{p_0, p_1, ..., p_n\}$ be a sequence of $n+1$ real numbers then the function (fct) p defined by

$$p(x) = p_0 + p_1\, x + ... + p_n\, x^n = p_0 + \sum_{k=1}^{n} p_k\, x^k$$

is called a polynomial (poly) of degree (deg) n.

1. $p_k \;=\; \frac{p^{\{k\}}(0)}{k!} \;\rightarrow\; p^{\{k\}}(0) \;=\; k!\, p_k$
2. $p(x)^2 = p_0^2 +\, (2\, p_0\, p_1)x +\, (2\, p_0\, p_2 + p_1^2)x^2 +$ (what goes here?) $+\, (2\, p_n\, p_{n-1})x^{2n-1} +$ $p_n^2 x_{2n}$
3. Let $q(x) = q_0 + q_1\, x + ... + q_n\, x^n = q_0 + \sum_{k=1}^{n} q_k\, x^k$ then

$$p(x)\, q(x) = p_0\, q_0 + \; (p_0\, q_1 + q_0\, p_1)x + \sum_{k=0}^{2} p_k\, q_{2-k}\, x^2 + \; \text{(what goes here?)}$$
$$+\, (p_{n-1}\, q_n + q_{n-1}\, p_n)x^{2n-1} + p_n\, q_n x^{2n}$$

DEF Let $P = \{p_0, p_1, ...\}$ be a sequence of real numbers then the function (fct) p defined by

$$p(x) = p_0 + p_1\, x + ... + p_n\, x^n + ... = p_0 + \sum_{k=1}^{\infty} p_k\, x^k$$

is called a power series in x.

1. $p_k \;=\; \frac{p^{\{k\}}(0)}{k!} \;\rightarrow\; p^{\{k\}}(0) \;=\; k!\, p_k$
2. $p(x)^2 = p_0^2 + \; (2\, p_0\, p_1)x + \; (2\, p_0\, p_2 + p_1^2)x^2 + ... +$
   $2 \sum_{k=0}^{i} p_k\, p_{2i+1-k}\, x^{2i+1} + 2 \sum_{k=0}^{i-1} (p_k\, p_{2i-k} + p_n^2)\, x^{2i} + ...$
3. Let $q(x) = q_0 + q_1\, x + ... + q_n\, x^n + ... = q_0 + \sum_{k=1}^{\infty} q_k\, x^k$ then

$$p(x)\, q(x) = p_0\, q_0 + \; (p_0\, q_1 + q_0\, p_1)x + ... + \sum_{k=0}^{n} p_k\, q_{n-k}\, x^n + ...$$

The values of x for which p is defined (converges to a number) is called the interval of convergence of p.

Ratio Test

If $\lim_{k\to\infty} |\frac{p_{k+1}}{p_k}| = r$ (exists as a number) then the interval of convergence for p is

$$(-\frac{1}{r}, \frac{1}{r});\ r \neq 0\ (-\infty, \infty);\ r = 0$$

(end values of the interval of convergence must be checked).