

Presentation Problems 4

1. In this problem, you will study the function $y = f(x) = \ln(x + 1)$ through its power series. Give (a) the interval of convergence of the power series for y (b) the values of the derivatives of y at $x = 0$ (c) the power series for $w = \frac{f(x)-f(0)}{x}$ and its interval of convergence (d) a comparison of the power series for y and w . Can you give a series for $\ln 2$ from (a)-(d)?
2. In this problem, you will study the function $y = f(x) = \ln(1 - x)$ through its power series. Give (a) the interval of convergence of the power series for y (b) the values of the derivatives of y at $x = 0$ (c) the power series for $w = \frac{f(x)-f(0)}{x}$ and its interval of convergence (d) a comparison of the power series for y and w . Can you give a series for $\ln 2$ from (a)-(d)?
3. In this problem, you will study the function $y = f(x) = \arctan(x)$ through its power series. Give (a) the interval of convergence of the power series for y (b) the values of the derivatives of y at $x = 0$ (c) the power series for $w = \frac{f(x)-f(0)}{x}$ and its interval of convergence (d) a comparison of the power series for y and w . Can you give a series for π from (a)-(d)?
4. In this problem, you will study the function $y = f(x) = e^x$ through its power series. Give (a) the interval of convergence of the power series for y (b) the values of the derivatives of y at $x = 0$ (c) the power series for $w = \frac{f(x)-f(0)}{x}$ and its interval of convergence (d) a comparison of the power series for y and w . Can you give a series for e from (a)-(d)?

Please use the ratio test and L'Hospital's rule. Also, give patterns that arise in your power series and the derivatives at $x = 0$.