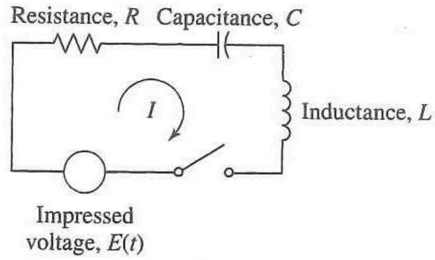


Figure 4.8

Another area where second order linear constant coefficient differential equations arise is in the flow of electric current in a circuit, as in Figure 4.8. The resistance, R, is measured in ohms; the capacitance, C, is measured in farads; and the inductance, L, is measured in henrys. We assume that all of these are constant values. The applied voltage, $E(t)$, is measured in volts and can change over time. We let I be the current in the circuit. If Q is the charge (measured in coulombs) on the capacitor, then

$$\frac{dQ}{dt} = I.$$

The flow of the current in the circuit is governed by Kirchoff's second law, which states: "In a closed circuit, the applied voltage is equal to the sum of the voltage drops across the circuit." Voltage drops are determined as follows:

The voltage drop across the resistor is IR.

The voltage drop across the capacitor is Q/C.

The voltage drop across the inductor is $L\frac{dI}{dt}$.

Combining these with Kirchoff's law, we obtain

$$LI' + RI + \frac{Q}{C} = E(t).$$

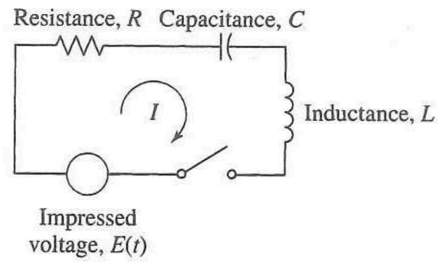
Differentiating this equation and using the fact that

$$\frac{dQ}{dt} = I,$$

gives us

$$\boxed{LI'' + RI' + \frac{1}{C}I = E'(t),}$$

which is a second order differential equation in I. If we also are given the initial charge and initial current or the initial current and the initial current's derivative, we have an initial value problem. Exercises 17–18 ask you to solve such initial value problems.



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