The One Dimensional Linear Advection-Diffusion Equation

James S. Sochacki
Department of Mathematics
James Madison University
jim@math.jmu.edu

This document gives an outline of the software modules that allow you to do a numerical study of the advection diffusion equation

$$\text{(AD)}\ w_t + cw_x + dw_{xx} = 0, -L < x < L, 0 < t < T$$

with initial condition

$$w(x, 0) = q \arctan(sx) + r$$

using the two difference techniques upwind and leap frog.

In the software modules described in this document you input c,d,L,a_left (the value for $w(x, 0)$ at $-\infty$), a_right (the value for $w(x, 0)$ at ∞) and T. You also input the time step, dt for t and the grid size, dx for x.

There are Maple routines, Matlab routines and Fortran 90 codes for running the numerical algorithms. The Matlab routines allow choices for the type of output to view the data. The Fortran 90 codes generate time frame data sets that can be viewed using the Maple or Matlab codes on the Fortran modules page. Follow the links to the type of software module you want to use.

To learn how to use the above routines run the upwind and leap frog codes using c = 1, d = 0, L = 10, a_left = -1, a_right = 1, s = 1, T = 10, dt = 0.25 and dx = 0.25.

Convince yourself using graphics that the results you are getting are similar to those predicted by the solutions to the advection diffusion equation for these parameters. The Maple and Matlab modules will do the graphics for you. The Fortran 90 codes output data sets. These data sets have the form name00010. You choose name. The 00010 represents the 10th time step. For the Fortran 90 codes you have to load the resulting data sets into a graphics visualizer. You can use either the Maple or Matlab graphics codes on the Fortran software module page. Click on the the Matlab Software Modules link to get the Matlab codes for the advection diffusion equation. Click on the the Maple Software Modules link to get the Maple codes for the advection diffusion equation. Click on the the Fortran Software Modules link to get the Fortran 90 codes for the advection diffusion equation.

Now let c = -1, d = 0, L = 10, a_left = -1, a_right = 1, T = 10, s = 1, dt = 0.25 and dx = 0.25. Compare the results with the previous input.

Now make L larger in the first input and see what happens. Make s larger and see what happens. Now make dt larger and see what happens. Then make dt smaller and see what happens. You should ask how these tests relates to the stability of the numerical algorithm.

You should then set dt back to 0.25 and choose d = 0.1 and d = -0.1. Consider the graphics in both these cases. Now decrease dt and see what happens. Are the results reasonable?

Now experiment on your own by changing c,d and dt. What results do you find? Can you explain these physically and numerically? Change the codes to run the initial condition

$$w(x,0) = exp(-x^2).$$

Run the parameters in this lesson and see what results you obtain.

Write a program in one of Maple, Matlab or Fortran 90 for the system of one dimensional advection equations

$$u_t + c_u v_x = 0 \ ; \ -L < x < L \ ; \ 0 < t < T$$
$$v_t + c_v u_x = 0 \ ; \ -L < x < L \ ; \ 0 < t < T$$

with initial conditions

$$u(x,0) = e^{-x^2} \ ; \ -L < x < L$$
$$v(x,0) = 0 \ ; -L < x < L$$

using upwind and leap frog on each equation. You should modify the existing codes. Run the codes for the parameters in this module. What do you discover?