EXAM I **Friday, Apr. 23** **NAME:**

FORMULAE:

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{x} = \sum_{k=0}^{\infty} \frac{x^k}{k!} \quad \text{for all x}$$

$$\log(1-x) = -\sum_{k=1}^{\infty} \frac{x^k}{k} \quad \text{for } x \in [-1,1)$$

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k \quad \text{for } x \in (-1,1)$$

$$\frac{x}{(1-x)^2} = \sum_{k=1}^{\infty} kx^k \quad \text{for } x \in (-1,1)$$

$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1} \quad \text{for all x}$$

$$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^{2k}}{(2k)!} x^{2k} \quad \text{for all x}$$

THEOREM 1: Consider the first order, initial value problem

$$y' = f(x,y), \quad y(x_0) = y_0,$$

and a rectangle, R, in the xy-plane such that $(x_0, y_0) \in R$. If f and $\frac{\partial f}{\partial y}$ are continuous on R, then there exists an interval, I, centered at x_0, and a unique solution $y(x)$ on I such that y satisfies the above initial value problem.

THEOREM 2: Consider the second order, linear, initial value problem

$$y'' + p(x)y' + q(x)y = g(x), \quad y(x_0) = y_0, \quad y'(x_0) = y_0',$$

where p, q, and g are continuous on an open interval, I, such that $x_0 \in I$. Then there exists a unique solution $y(x)$ on I such that y satisfies the above initial value problem.