

DIRECTIONS:

- **STAPLE** this page to the front of your homework (don't forget your name!).
- Show all work, clearly and in order **You will lose points if you work is not in order.**
- When required, **do not forget the units!**
- Circle your final answers. **You will lose points if you do not circle your answers.**

Question	Points	Score
1	1	
2	1	
3	4	
4	1	
5	1	
6	2	
Total	10	

Problem 1: After reading “A Guide to Writing Mathematic” by Dr. K. P. Lee, list five items pertaining to writing math of which you were previously unaware. (If you were in math 200 with me, write down five things you learned from writing your reports).

Answers will vary.

Problem 2: (1 points) Write down a differential equation of the form $y' = ay + b$ whose solutions approach $y = \frac{2}{3}$ as $t \rightarrow \infty$.

Consider the equation

$$y' = -3y + 2.$$

Then if we look at the “slopes” on the line $y = \frac{2}{3} \pm \delta y$ in the $y - t$ plane, we see that the flow lines will tend towards $y = \frac{2}{3}$ as $t \rightarrow \infty$.

Problem 3: (4 points) A pond initially contains 1,000,000 gallons of water and an unknown amount of an undesirable chemical. Water containing 0.01 grams of this chemical per gallon flows into the pond at a rate of 300 gal/h. The mixture flows out at the same rate, so the amount of water in the pond remains constant. Assume that the chemical is uniformly distributed throughout the pond.

(a) (2 point) Write down a differential equation for the amount of chemical in the pond at any time.

Let c be the amount of chemical in the pond (in grams) and let t be time (in hours). Then the differential equation is given by

$$\frac{dc}{dt} = 0.01 \text{ g/gal} \cdot 300 \text{ gal/hr} - c \text{ g} \times 10^{-6} / \text{gal} \cdot 300 \text{ gal/hr}.$$

Hence

$$\frac{dc}{dt} = 3(1 - 10^{-4}c).$$

(b) (2 point) How much of the chemical will be in the pond after a very long time? Does this limiting amount depend on the amount that was present initially?

As $t \rightarrow \infty$, then we see $c \rightarrow 10^4$ grams, which does not depend on the initial amount present.

Problem 4: (1 point) Solve the initial value problem given by $y' = 2y - 5$ where $y(0) = y_0$.

Consider

$$y' = 2 \left(y - \frac{5}{2} \right).$$

Then by separation of variables

$$\frac{dy}{y - \frac{5}{2}} = 2dt,$$

and by integrating both sides

$$\log |y - 5/2| = 2t + c.$$

Using the initial condition $y(0) = y_0 \implies c = \log |y_0 - 5/2|$ so the solution is

$$y = \left(y_0 - \frac{5}{2} \right) e^{2t} + \frac{5}{2}.$$

Problem 5: (1 point) Solve the initial value problem given by $y' + 2y = te^{-2t}$ where $y(1) = 0$.

The integration factor, $\mu(t)$, is given by

$$\frac{d\mu}{dt} = 2\mu,$$

hence by separation of variables $\mu = e^{2t}$. This implies that

$$\frac{d}{dt} (e^{2t}y) = e^{2t} (te^{-2t}) = t.$$

Hence

$$y = e^{-2t} (t^2 + c).$$

Using the initial condition $y(1) = 0 \implies c = -1$, so the solution is

$$y = e^{-2t} (t^2 - 1).$$

Problem 6: (2 points) Solve the following and find the value of y_0 for which the solution of the initial value problem

$$y' - y = 1 + 3 \sin t, \quad y(0) = y_0,$$

remains finite.

By the same procedure as the last problem, the integration factor is $\mu(t) = e^{-t}$, therefore

$$e^{-t}y = \int_{t_0}^t e^{-s} (1 + 2 \sin s) ds = -e^{-t} \cos t - 3 \int_{t_0}^t e^{-t} \sin t dt.$$

Using integration by parts twice we find

$$\int e^{-t} \sin t dt = -\frac{1}{2}e^{-t} (\sin t + \cos t),$$

hence

$$y = -1 - \frac{3}{2} (\sin t + \cos t) + ce^t.$$

Using the initial condition $y(0) = y_0$ implies $c = y_0 + \frac{5}{2}$. Looking at the solution, we see that if $c \neq 0$ then the solution is unbounded as $t \rightarrow \infty$. Therefore, the solution will remain finite only if $c = 0 \implies y_0 = -\frac{5}{2}$.