HOMEWORK 2 **DUE: Fri., Apr. 9** **NAME:**

DIRECTIONS:

- **STAPLE** this page to the front of your homework (don't forget your name!).
- Show all work, clearly and in order **You will lose points if you work is not in order**.
- When required, **do not forget the units**!
- Circle your final answers. **You will lose points if you do not circle your answers**.

Question	Points	Score
1	3	
2	3	
3	1	
4	2	
5	1	
Total	10	

Problem 1: (3 points) Are the following differential equations linear or nonlinear? What order are they? What techniques would you use to solve them?

(a) (1 point) $\sin^2 xy' = \arctan x e^{\frac{x^3}{2}} y + 100\pi \frac{x^4}{x^2+1}$.

Linear, first order. You might use the integration factor discussed in section 2.1.

(b) (1 point) $(1 + \cos x)\, dy = (e^{-y} + 1) \sin x dx$.

Nonlinear, first order. This equation is separable.

(c) (1 point) $2x + y^2 + 2xyy' = 0$.

Nonlinear, first order. Use the integrating factors discussed in section 2.6.

Problem 2: (3 points) Find a family of solutions to

$$\frac{dy}{dx} = xy^{1/2},$$

using separation of variables. Now find a particular solution to this problem subject to $y(0) = 0$.

By using separation of variables we obtain

$$\frac{dy}{y^{1/2}} = xdx.$$

Integrating both sides yields

$$2y^{1/2} = \frac{x^2}{2} + c_1.$$

The family of solutions is given by solving for y

$$y = \left(\frac{x^2}{4} + C\right)^2.$$

$C = 0$ satisfies the initial condition so the particular solution is $y = \frac{x^4}{16}$.

Problem 3: (1 points) Consider the solution you found from problem 2 above. Does the family of solutions you found represent all solutions to this problem? Support your answer.

No, for example, the infinite family

$$y = \begin{cases} 0, & x < a, \\ \frac{(x^2-a^2)^2}{16}, & x \geq a, \end{cases}$$

for any $a \geq 0$ are also solutions, which are not members of the family described above.

Problem 4: (2 points) Is the particular solution you found in problem 2 above unique? If not, why does this not contradict the following theorem guaranteeing uniqueness:

Theorem Consider the initial value problem $y' = f(x, y)$ subject to $y(x_0) = y_0$, and a rectangle, R, in the xy-plane such that $(x_0, y_0) \in R$. If f and $\frac{\partial f}{\partial y}$ are continuous on R, then there exists an interval, I, centered at x_0, and a unique solution $y(x)$ on I such that y satisfies the initial value problem.

No. For example, $y = 0$ is a solution of the differential equation in problem 2 and satisfies the initial condition $y(0) = 0$ but is not a member of the family of solutions described above. This does not contradict the theorem above since $f(x, y) = xy^{1/2}$ and $\frac{\partial f}{\partial y} = \frac{x}{2y^{1/2}}$ both do not exist for $y < 0$. Hence they cannot be continuous in a rectangle R with $(x_0, y_0) = (0, 0) \in R$. Therefore the theorem doesn't apply and we cannot guarantee uniqueness of the solution to the initial value problem

Problem 5: (1 point) List your project group member names (including your own), email addresses, and phone numbers. Remember groups must consist of 3-4 people.