HOMEWORK 4 **DUE: Friday, Jan. 29** **NAME:**

DIRECTIONS:

- **STAPLE** this page to the front of your homework (don't forget your name!).
- Show all work, clearly and in order **You will loose points if you work is not in order**.
- When required, **do not forget the units**!
- Circle your final answers. **You will loose points if you do not circle your answers**.

Question	Points	Score
1	1	
2	2	
3	3	
4	3	
5	1	
Total	10	

Problem 1: (1 point) Calculate the second-order Taylor polynomial for $f(x, y) = \cos x \cos y$ at the point $(0, \pi/2)$.

Problem 2: (2 points) A metal plate has the shape of the region $x^2 + y^2 \leq 1$. The plate is heated so that the temperature at any point (x, y) on it is indicated by

$$T(x, y) = 2x^2 + y^2 - y + 3.$$

Fine the hottest and coldest points on the plate, and the temperature at each of these points (Hint: Parametrize the boundary of the plate in order to find any critical points there.)

Problem 3: (3 points) Suppose the cone $z^2 = x^2 + y^2$ is sliced by the plane $z = x + y + 2$ so that a conic section C is created. Use Lagrange multipliers to find the points on C that are nearest to and farthest from the origin. (Hint: Think about the shape of C. What does it look like?).

Problem 4: (3 points) Find the critical points of $f(x, y) = x^2 + y$ subject to $x^2 + 2y^2 = 1$ and use the Hessian criterion to determine the nature of the critical point(s).

Problem 5: (1 point) Consider the equations that relate polar and Cartesian coordinates:

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}$$

These equations define x and y as functions or r and θ. Use the Inverse Function theorem to determine the set of points $\{\mathbf{x}\}$ near which we can invert these equations. What can you say about the inverse function theorem at the origin?