

DIRECTIONS:

- **STAPLE** this page to the front of your homework (don't forget your name!).
- Show all work, clearly and in order **You will lose points if you work is not in order.**
- When required, **do not forget the units!**
- Circle your final answers. **You will lose points if you do not circle your answers.**

Question	Points	Score
1	3	
2	2	
3	1	
4	2	
5	2	
Total	10	

Problem 1: (3 point) Consider the differential equation

$$y^{(4)} + 2y''' + y'' = 0.$$

Verify that the functions $y_1(t) = 1$, $y_2(t) = t$, $y_3(t) = e^{-t}$, and $y_4(t) = te^{-t}$ are solutions to the equation and determine their Wronskian.

Problem 2: (2 points) Let the linear differential operator L be defined by

$$L[y] = a_0y^{(n)} + a_1y^{(n-1)} + \dots + a_ny,$$

where $a_0, a_1, \dots, a_n \in \mathcal{R}$.

(a) (1 point) Find $L[t^n]$.

(b) (1 point) Find $L[e^{mt}]$.

Problem 3: (1 point) Determine four solutions of the equation $y^{(4)} - 5y'' + 4y = 0$. Do you think the four solutions form a fundamental set of solutions? Explain your answer.

Problem 4: (2 points) Find the general solution to the differential equation

$$y^{(5)} + 5y^{(4)} - 2y^{(3)} - 10y^{(2)} + y^{(1)} + 5y = 0.$$

Problem 5: (2 point) Solve the initial value problem

$$y''' - 8y = 0,$$

subject to $y(0) = 0$, $y'(0) = -1$, $y''(0) = 0$.