

DIRECTIONS:

- **STAPLE** this page to the front of your homework (don't forget your name!).
- Show all work, clearly and in order **You will lose points if you work is not in order.**
- When required, **do not forget the units!**
- Circle your final answers. **You will lose points if you do not circle your answers.**

Question	Points	Score
1	1	
2	2	
3	2	
4	3	
5	2	
Total	10	

Problem 1: (1 point) Find the values of $i\sqrt{3}$ (Note, you needn't restrict yourself to the Principle Branch).

Problem 2: (2 points) Find the solutions of

$$z^{\pi+i} = 10.$$

Problem 3: (2 points) Given the formulae

$$\cosh x = \frac{1}{2} (e^x + e^{-x}), \text{ and } \sinh x = \frac{1}{2} (e^x - e^{-x}),$$

for $x \in \mathcal{R}$, prove that

$$\cos z = \cos x \cosh y - i \sin x \sinh y.$$

for $z = x + iy \in \mathcal{C}$.

Problem 4: (3 points) Prove that

$$\tan^{-1} z = \frac{i}{2} \text{Log} \left(\frac{1-iz}{1+iz} \right),$$

for $z \neq \pm i$.

Problem 5: (2 points) Parametrize the circle of radius ϵ centered at $z_0 \in \mathcal{C}$. How does this relate to proving that

$$\int_{\gamma} \frac{1}{z - z_0} = \begin{cases} 0, & \text{if } z_0 \text{ is outside } \gamma, \\ 2\pi i, & \text{if } z_0 \text{ is inside } \gamma. \end{cases}$$

where γ is a piece-wise smooth, positively oriented, simple closed curve.