HOMEWORK 9 **DUE: Fri., May 28** **NAME:**

DIRECTIONS:

- **STAPLE** this page to the front of your homework (don't forget your name!).
- Show all work, clearly and in order **You will lose points if you work is not in order**.
- When required, **do not forget the units**!
- Circle your final answers. **You will lose points if you do not circle your answers**.

Question	Points	Score
1	2	
2	1	
3	4	
4	3	
Total	10	

Problem 1: (2 points) Recall that a function $f(z) = u + iv$ is analytic at a point z_0 if it is analytic in some neighborhood about z_0. That is, there exists some $R > 0$ such that f is analytic on $B_R(z_0) = \{z \in \mathbb{C} || z - z_0| < R\}$. Show that the function $f(z) = (x^2 + y) + i(y^2 - x)$ is not analytic at any point. (Hint: recall that if f is analytic in an open set, then the Cauchy-Riemann equations must hold on that set.)

Problem 2: (1 point) Find the radius of convergence of the power series

$$\sum_{k=0}^{\infty} \frac{z^{3k}}{2^k}.$$

(Hint, you can't use either the ratio test or the root test directly.)

Problem 3: (4 points) Consider the complex valued function

$$f(z) = \frac{z^3 + 1}{(z - i)^3}.$$

(a) (1 point) Is the point $z_0 = i$ a removable singularity or a pole?

(b) (2 points) Find the Laurent series for f about $z_0 = i$.

(c) (1 point) Calculate

$$\text{Res}(f; z_0) = \frac{1}{2\pi i} \int_{|\zeta - z_0| = s} f(\zeta) d\zeta.$$

Problem 4: (3 points) Consider the complex valued function

$$f(z) = \frac{z^3 + z^2}{(z-1)^2}.$$

Let γ_1 be a parametrization of the circle $|z| = 3$ and γ_2 be a parametrization of the circle $|z - 4| = 2$.

(a) (1 point) Draw γ_1 and γ_2 and plot the singularities of f.

(b) (2 points) Calculate

$$\int_{\gamma_1} f(\zeta) d\zeta,$$

and

$$\int_{\gamma_2} f(\zeta) d\zeta.$$

(Hint, think about using the Residue Theorem.)