Romeo and Juliet, a Dynamical Love Affair

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Abstract

We model the oscillating emotions felt by Romeo and his fair Juliet as they feed on each others passion and indifference in turn. We model this emotional situation using simple, linear differential equations in the style of Strogatz [1, 2]. Using geometric and computational approaches, we observe that this situation results in the never ending cycle of ardor and indifference.

1 Problem Statement

Love and lovers are fickle and moody creatures which both crave and flee from pursuit. For example, when our tragic hero and heroin first take the stage together in act 1, scene 5 of Shakespeare's *Romeo and Juliet* [3], they are both attracted to the *beautiful unknown.*

> **Romeo:** *O, she doth teach the torches to burn bright!*
> *It seems she hangs upon the cheek of night*
> *like a rich jewel in an Ethiope's ear*
> *Beauty too rich for use, for earth too dear!*
> *So shows a snowy dove trooping with crows,*
> *As yonder lady 0'er her fellows shows.*
> *The measure done, I'll watch her place of stand, And, touching hers, make blessed my rude hand.*
> *Did my heart love till now? Forswear it, sight!*
> *For I ne'er saw true beauty till this night.*

(missing scenes)

> **Romeo:** *If I profane with my unworthiest hand*
> *This holy shrine, the gentle fine is this:*
> *My lips, two blushing pilgrims, ready stand*
> *To smooth that rough touch with tender kiss.*
>
> **Juliet:** *Good pilgrim, you do wrong your hand too much,*
> *Which mannerly devotion show in this;*
> *For saints have hands that pilgrim's hands do touch,*
> *And palm to palm is holy palmer's kiss.*
>
> **Romeo:** *Have not saints lips, and holy palmers too?*
>
> **Juliet:** *Ay, pilgrim, lips that they must use in prayer.*

Here Romeo expresses his love (a.k.a. interest), and this catches Juliet's eye. However, he goes on with a little too much ardor pressing for a kiss, cooling her interest. Then he backs off slightly (only slightly, that dog!) and Juliet warms up again, to which he intern warms up.

Romeo: *O, then, dear saint, let lips do what hands do;*
They pray, grant thou, lest faith turn to despair.

Juliet: *Saints do not move, though grant for prayer's sake.*

Romeo: *Then move not, while my prayer's effect I take.*
Thus from my lips, by yours, my sin is purged.

Here they kiss, and the cycle continues. Would this cycle of "hot and cold" have continued if our star crossed lovers had not died in the play?

2 Model Design

Let us define our variables as follows. Let $R(t)$ be Romeo's feelings for Juliet at time t and $J(t)$ be Juliet's feelings for Romeo at time t. Let us suppose that positive values of R and J signify love, passion, and attraction, while negative values signify dislike, and $R = J = 0$ signifies indifference.

2.1 Romeo, Romeo...

In the simple system we initially proposed, Juliet's change in feelings depend only on Romeo's current feelings and visa-versa. Then we may proposed a model as follows

$$\frac{dR}{dt} = aJ, \tag{2.1}$$

$$\frac{dJ}{dt} = -bR, \tag{2.2}$$

where the parameters a and b are real, positive numbers and have unites of 1/time (so both sides of the equations have units of love/time). Then the only time when their feelings are constant, is when equations 2.1-2.2 are zero. That is when

$$\frac{dR}{dt} = 0,$$

$$\frac{dJ}{dt} = 0.$$

But this implies that $J(t) = R(t) = 0$ for all time. At any other value the lovers emotions will cycle from hot to cold to hot again indefinitely as is shown in Section 3.1.

2.2 A More General System

In general, it is possible that unlike our Romeo and Juliet, the lovers in question are at least somewhat self aware and the change in emotions might also be "self-monitering." That is, the equations might be of the form

$$\frac{dR}{dt} = aR + bJ, \tag{2.3}$$

$$\frac{dJ}{dt} = cR + dJ. \tag{2.4}$$

Since this is still a linear system, the only solution is still $J(t) = R(t) = 0$, however, we may find that depending on the values of the parameters and initial conditions (i.e. initial feelings towards one another) the result could be a variety of conditions. What can you say about the stability of the steady state or the long term behavior or Romeo and Juliet for a given set of initial emotions?

3 Implementation and Results

To look at this system, we see that there are no null clines (since the system is linear) but we can plot the fixed points and some trajectories. We can also consider the eigen values of the system to determine the stability of the fixed points.

3.1 Romeo, Romeo...

For equations 2.1-2.2 we see that the system can be rewritten as

$$\begin{pmatrix} \frac{dR}{dt} \\ \frac{dJ}{dt} \end{pmatrix} = \begin{pmatrix} 0 & a \\ -b & 0 \end{pmatrix} \begin{pmatrix} R \\ J \end{pmatrix}. \tag{3.1}$$

The eigen values of this matrix (note, this is a diagonal matrix) are $\lambda_{1,2} = \pm\sqrt{ab}i$ and the solutions are of the form

$$J(t) = c_1 e^{\lambda_1} + c_2 e^{\lambda_2}.$$

Since the eigen values are purely complex, this means that our fixed point is either a cycle or a spiral. Using Eulers formula ($e^{i\theta} = \cos\theta + i\sin\theta$ for $\theta \in \mathbb{R}$) we see that since the real part of $\lambda_{1,2}$ is/are zero, the steady state at $J(t) = R(t) = 0$ is neutrally stable, indicating it must be a center. Looking at figure 1 we see that we do, indeed see the lovers emotions in perpetual oscillation.

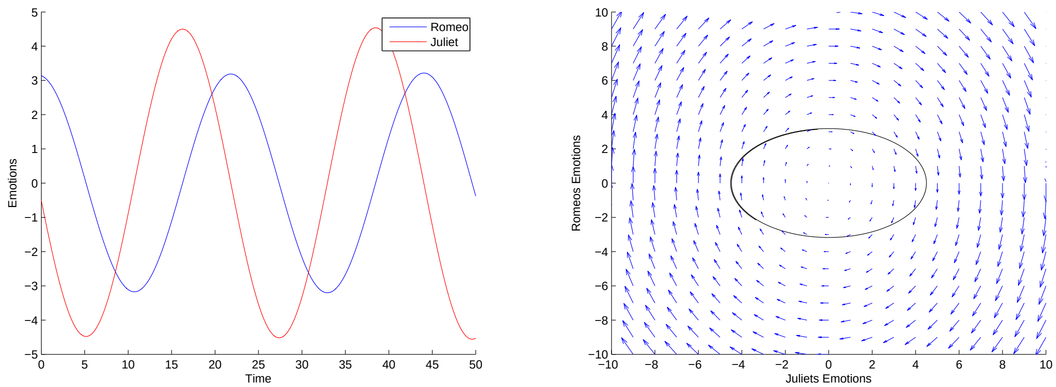

Figure 1: Figure 1: (Left) In this figure we have plotted the solutions of equations 2.1-2.3 for initial values of $R(0) = 3.14$ and $J(0) = -0.5$ with $a = 0.2$ and $b = 0.4$ and (Right) we have plotted the phase portrait along with the solution for these same initial conditions and parameter values.

3.2 A More General System

What is the stability of the fixed point here? What are the eigen values?

4 Conclusion

What are the emotional interpretations of the parameters a, b, c, and d? How do the solutions, steady states (and their stabilities), as well as the phase portraits of the "Romeo, Romeo" case differ from the more general situation? What can you say about real romances? Can you expand this model to predict the behavior and durability of celebrity romances?

References

[1] Strogatz, S. H. (1988) Love affairs and differential equations. *Math. Magazine* **61**, 35.

[2] Strogatz, S. H. (1994) Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering. Perseus Books Publishing, LLC.

[3] Shakespeare, W. (1597) Romeo and Juliet.