HOMEWORK 2 **DUE: Fri., Apr. 6** **NAME:**

DIRECTIONS:

- Turn in your homework as **SINGLE-SIDED** typed or handwritten pages.
- **STAPLE** your homework together. Do not use paper clips, folds, etc.
- **STAPLE** this page to the front of your homework.
- Be sure to write your name on your homework.
- Show all work, **clearly and in order**.

You will lose point 0.5 points for each instruction not followed.

Questions	Points	Score
1	1	
2	1	
3	1	
4	2	
5	3	
6	1	
7	1	
Total	10	

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Problem 1: (1 point) Suppose $A \neq \emptyset$ and $B \neq \emptyset$. Show that $A \times B = B \times A$ iff $A = B$.

Problem 2: (1 point) If A, B, and C are finite sets, show that

$$^{\#}(A \cup B \cup C) =^{\#} A +^{\#} B +^{\#} C -^{\#} (A \cap B) -^{\#} (A \cap C) -^{\#} (B \cap C) +^{\#} (A \cap B \cap C).$$

Problem 3: (1 point) If a, $b \in \mathbb{Z}$, show $(-a)(-b) = ab$.

Problem 4: (2 points) If a, $b \in \mathbb{Z}$,

(a) (1 point) Suppose $0 < a$ and $0 < b$. Show that $a < b$ iff $a^2 < b^2$.

(b) (1 point) Suppose $a < 0$ and $b < 0$. Show that $a < b$ iff $b^2 < a^2$.

Problem 5: (3 points) If n, k are non-negative integers, we define the binomial coefficient, $\binom{n}{k}$, by

$$\binom{n}{k} = \frac{n!}{k!(n-k)!},$$

where $n! = n \cdot (n-1) \cdots 2 \cdot 1$, and we set $0! = 1$.

(a) (2 points) Prove that

$$\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r},$$

for $r = 1$, 2, 3,..., n

(b) (1 points) Using part (a), prove the Binomial Theorem:

If a, $b \in \mathbb{Z}$ and n is a positive integer, then

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}.$$

Hint: Use mathematical induction

Problem 6: (1 point) Let n be an integer greater than or equal to 2. If $a, b \in \mathbb{Z}$, we say that $a \sim b$ iff $a - b$ is a multiple of n, that is, n divides $a - b$. Prove this defines an equivalence relation.

Problem 7: (1 point) Let n be a positive integer greater than or equal to 2. Then there exists a prime p such that p divides n.

Hint: Consider using the Principle of Strong Induction: To prove an infinite sequence of statements $p(n)$ for $n = b, b+1, \ldots$, prove the following implication for $k = b, b+1, b+2, \ldots$: $p(m)$ for all m such that $b \leq m < k \Longrightarrow p(k)$.