HOMEWORK 4 **DUE: Fri., Apr. 20** **NAME:**

DIRECTIONS:

- Turn in your homework as **SINGLE-SIDED** typed or handwritten pages.
- **STAPLE** your homework together. Do not use paper clips, folds, etc.
- **STAPLE** this page to the front of your homework.
- Be sure to write your name on your homework.
- Show all work, **clearly and in order**.

You will lose point 0.5 points if one or more of these instructions are not followed.

Questions	Points	Score
1	2	
2	2	
3	2	
4	2	
5	2	
Extra Credit 1	2	
Extra Credit 2	1	
Extra Credit 3	2	
Extra Credit 4	1	
Total	10	

Problem 1: (2 points) Show that the composition of bijections is a bijection.

Problem 2: (2 points) If A is finite and $x \notin A$, then $A \cup \{x\}$ is finite and $Card\,(A \cup \{x\}) = Card\,(A) + 1$.

Problem 3: (2 points) If B is a finite set and $A \subseteq B$ then A is finite and $Card\,(A) \leq Card\,(B)$. *Hint: Use induction and problem 2.*

Problem 4: (2 points) If A is a subset of a countable set B, then A is countable. *Hint: Use problem 5.*

Problem 5: (2 points) So that if D is a denumerable set and $f : D \to A$ is onto, then there is a $g : A \to D$ such that g is 1-1.

Extra Credit 1: (2 points) Let A and B be sets and let $f : A \to B$ be a function. Suppose that $\{A_i\}_{i \in I}$ is a collection of subsets of A and $\{B_j\}_{j \in J}$ is a collection of subsets of B.

(a) (1 point) Show that $f\left(\cup_{i \in I} A_i\right) = \cup_{i \in I} f\left(A_i\right)$.

(b) (1 point) Suppose f is a bijection. Show that $f^{-1}\left(\cap_{i \in J} B_j\right) = \cap_{j \in J} f^{-1}\left(B_j\right)$.

Extra Credit 2: (1 points) For following functions, find $f(A)$ and $f^{-1}(B)$.

(a) (0.5 point) $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = \sin x$, $A = \{-2, -1, 0, 1, 2\}$, $B = \{0, 1, 2\}$.

(b) (0.5 point) $f : \mathbb{R} \to \mathbb{Z}$ is is the floor function defined by

$$f(x) = \lfloor x \rfloor = n$$

where $n \leq x < n + 1$ for $n \in \mathbb{N}$ and $A = (0, 5)$, $B = \{0, 1, 2\}$.

Extra Credit 3: (2 points) Let $f : A \to B$ and $B' \subseteq B$.

(a) (1 point) Prove that $f\left(f^{-1}(B')\right) \subseteq B'$.

(b) (1 point) Prove that if f is onto, then $f\left(f^{-1}(B')\right) = B'$.

Extra Credit 4: (1 point) Prove or find a counterexample to the following conjecture. Assume $f : X \to Y$ and $A, B \subset X$ If $f(A) \setminus f(B) = \emptyset$, then $f(A \setminus B) = \emptyset$.