

DIRECTIONS:

- Turn in your homework as **SINGLE-SIDED** typed or handwritten pages.
- **STAPLE** your homework together. Do not use paper clips, folds, etc.
- **STAPLE** this page to the front of your homework.
- Be sure to write your name on your homework.
- Show all work, **clearly and in order**.

You will lose point 0.5 points for each instruction not followed.

Questions	Points	Score
1	1	
2	2	
3	2	
4	3	
5	1	
6	1	
Total	10	

Problem 1: (1 point) Let a be a positive rational number. Let $A = \{x \in \mathbb{Q} \mid x^2 < a\}$. Show that A is bounded in \mathbb{Q} . Does it have a least upper bound?

Problem 2: (2 points) Let $\wp(X)$ be the power set of X . Define the binary relation on $\wp(X)$ as follows: $A, B \in \wp(X)$, $A \sim B \iff A \subseteq B$. Verify that $\wp(X)$ under this relation is a partially ordered set (poset).

Problem 3: (2 points) Prove that $\sqrt{2}$ is not a rational number.

Problem 4: (3 points) Prove that an ordered field has the least upper bound property if and only if it has the greatest lower bound property.

Problem 5: (1 point) Let $a, b \in \mathbb{N}$. We define a number $n \in \mathbb{N}$ to be even if $n = 2k$ for some $k \in \mathbb{N}$. Similarly, we define a number $n \in \mathbb{N}$ to be odd, if $n = 2k + 1$ for some $k \in \mathbb{N}$.

(a) (0.5 points) Prove that if a and b are odd, then $a \cdot b$ is also odd.

(b) (0.5 points) Prove that $a \cdot b$ is even if and only if a is even, b is even, or both are even.

Problem 6: (1 point) Let r be a rational number such that $r \neq 0$ and s be an irrational number.

(a) (0.5 points) Prove that $r + s$ is irrational.

(b) (0.5 points) Prove that $r \cdot s$ is irrational.